

Geometry of linear transformations

$\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$ Scaling	Sub-classes Dilation ($k > 1$) and Contraction ($0 < k < 1$).
$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ Projection	Define $\text{proj}_L(\mathbf{x}) = 2(\mathbf{x} \cdot \mathbf{u})\mathbf{u}$ where $\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ is a unit vector, $u_1^2 + u_2^2 = 1$. The matrix is $\begin{pmatrix} u_1u_1 & u_1u_2 \\ u_1u_2 & u_2u_2 \end{pmatrix}$
$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ Reflection	Define $\text{refl}_L(\mathbf{x}) = 2(\mathbf{x} \cdot \mathbf{u})\mathbf{u} - \mathbf{x}$. The matrix is $\begin{pmatrix} a & b \\ b & -a \end{pmatrix}$, $a^2 + b^2 = 1$.
$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ Rotation	In general, $\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$
$\begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix}$ Scaled Rotation	In general, $\begin{pmatrix} r \cos \theta & r \sin \theta \\ -r \sin \theta & r \cos \theta \end{pmatrix}$
$\begin{pmatrix} 1 & 0 \\ k & 1 \end{pmatrix}$ Vertical Shear	Change vertical $y \rightarrow y + kx$, leave x fixed.
$\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$ Horizontal Shear	Change horizontal $x \rightarrow x + ky$, leave y fixed.