Linear Transformation

A linear transformation is a function T defined on a vector space V with range in a vector space W satisfying the rules
(a) $T(\mathrm{v}_1 + \mathrm{v}_2) = T(\mathrm{v}_1) + T(\mathrm{v}_2)$
(b) $T(k\mathrm{v}_1) = kT(\mathrm{v}_1)$.

Theorem 1 (Matrix of T)
Assume $V = R^n$ and $W = R^m$. Then T is represented as a matrix multiply

$$T(\mathrm{x}) = A\mathrm{x}$$

where A is the $n \times m$ matrix whose columns are given in terms of the identity matrix I and function T by the formula

$$\mathrm{col}(A, j) = T(\mathrm{col}(I, j)), \quad j = 1, \ldots, n.$$

Definition: A basis of a vector space V is a set of vectors $\mathrm{v}_1, \ldots, \mathrm{v}_n$ such that every vector v in V can be uniquely written as a linear combination of $\mathrm{v}_1, \ldots, \mathrm{v}_n$. Briefly, the vectors *span* V and are *independent*.

Theorem 2 (Representation of T)
Every basis $\{\mathrm{v}_1, \ldots, \mathrm{v}_n\}$ of V gives a relation

$$T\left(\sum_{j=1}^{n} c_j \mathrm{v}_j\right) = \sum_{j=1}^{n} c_j \mathrm{w}_j, \quad \text{where} \quad \mathrm{w}_j = T(\mathrm{v}_j).$$