Fundamental Theorem of Calculus

Isaac Newton found these formulas in an effort to extend the gas mileage formula Distance = Rate $\times$ Time [$D = RT$] to instantaneous rates.

$$\begin{aligned} &(a) \quad \int_a^b f'(x)dx = f(b) - f(a) \\ &(b) \quad \left(\int_a^x g(t)dt\right)' = g(x) \end{aligned}$$

Part (a) is used in differential equations in the alternative form, which uses indefinite integrals:

$$(a) \quad \int_a^b y'(x)dx = y(x) + C$$

Method of Quadrature

Also called the *integration method*, the idea is to multiply the differential equation by dx, then write an integral sign on each side.

- The method applies only to quadrature equations $y' = F(x)$.
- The Fundamental Theorem of Calculus is applied on the left side to evaluate $\int y'(x)dx$ as $y(x)$ plus a constant.
- The method finds a candidate solution $y(x)$. It does not verify that the expression works.

Example: Method of Quadrature

Solve by the method of quadrature $y' = 2x$.

- Multiply $y' = 2x$ by dx, then write an integral sign on each side.

$$\int y'(x)dx = \int 2xdx$$

- Apply the FTC $\int y'(x)dx = y(x) + C$ on the left:

$$y(x) + c_1 = \int 2xdx$$

- Evaluate the integral on the right by tables. Then

$$y(x) + c_1 = x^2 + c_2, \quad \text{or} \quad y(x) = x^2 + C$$