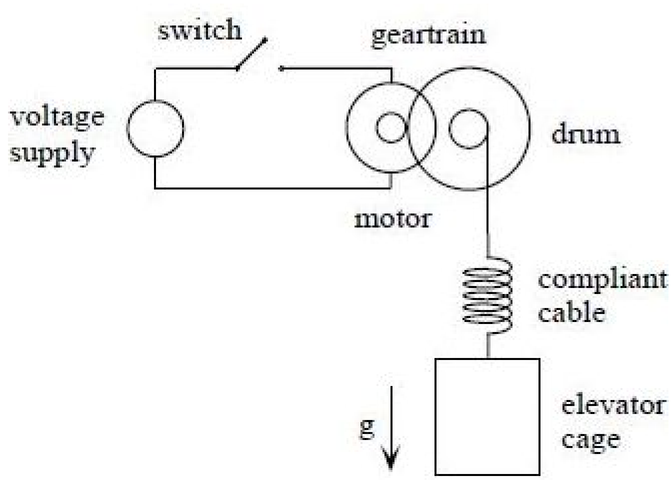
EXAMPLE: Vibration in a Cable Hoist

The Problem

The cage of an elevator is hoisted by a long cable wound over a drum driven through a gear-set by an electric motor.

The motor is relay-operated as either on or off and the resulting abrupt transients cause the cage to oscillate on the hoisting cable. Because the cable has low internal friction, then the oscillations persist for many cycles.

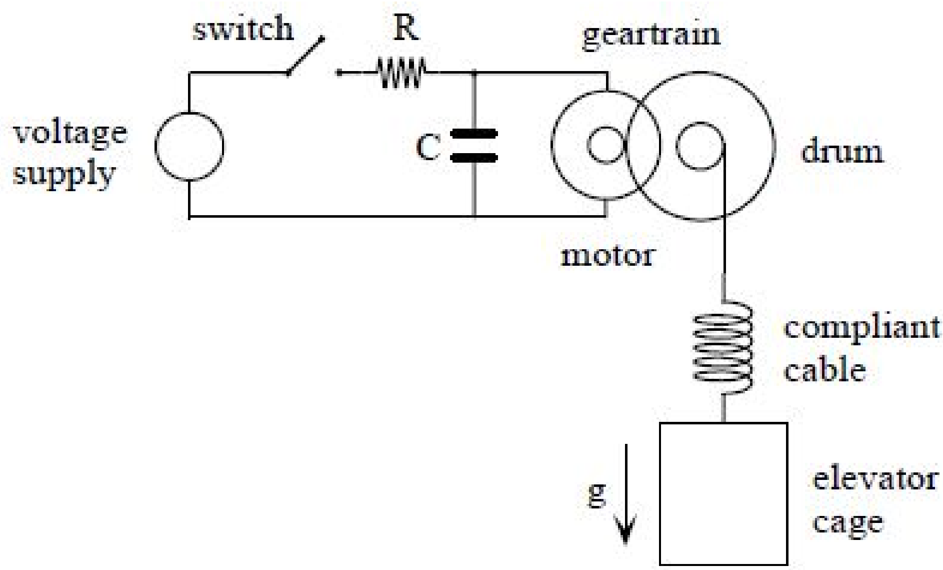
The peak stress in the cable is almost double the steady-state stress required to support the weight of the cable.



Modeling to Reduce Oscillations

A designer introduces an electrical R-C filter between the relay and the motor terminals. Will this change smooth the transient, and thereby reduce the oscillation amplitude? Some questions:

- Abruptly engaging the motor excites oscillation. Does electrical filtering change the oscillations?
- What are the simplest models which can show the effect of electrical filtering on the mechanical oscillation?



A Differential Equation Model

Newtonian mechanics:

$$\boldsymbol{m}_{\mathsf{cage}}\boldsymbol{x}''_{\mathsf{cage}} = \boldsymbol{k}_{\mathsf{cable}}(\boldsymbol{x}_{\mathsf{rim}} - \boldsymbol{x}_{\mathsf{cage}}) - \boldsymbol{m}_{\mathsf{cage}}\boldsymbol{g}$$

Drum:

$$\begin{aligned} \boldsymbol{x}'_{\mathsf{rim}} &= \boldsymbol{r}_{\mathsf{drum}}\boldsymbol{\omega}_{\mathsf{drum}} \\ \boldsymbol{\omega}_{\mathsf{drum}} &= \boldsymbol{r}_{\mathsf{drum}}\boldsymbol{F}_{\mathsf{cable}} \end{aligned}$$

Motor:

$$\begin{aligned} \boldsymbol{\omega}_{\mathsf{motor}} &= \boldsymbol{e}_{\mathsf{motor}}/\boldsymbol{K}_{\mathsf{motor}} \\ \boldsymbol{i}_{\mathsf{motor}} &= \boldsymbol{\tau}_{\mathsf{motor}}/\boldsymbol{K}_{\mathsf{motor}} \end{aligned}$$

Gear Train:

$$\begin{aligned} \boldsymbol{\omega}_{\mathsf{drum}} &= \boldsymbol{n}_{\mathsf{gear}}\boldsymbol{\omega}_{\mathsf{motor}} \\ \boldsymbol{\tau}_{\mathsf{motor}} &= \boldsymbol{n}_{\mathsf{gear}}\boldsymbol{\tau}_{\mathsf{drum}} \end{aligned}$$

Switch without RC-circuit and without electrical-mechanical power analysis:

$$\boldsymbol{e}_{\mathsf{motor}} = \begin{cases} \boldsymbol{e}_{\mathsf{supply}} & \text{switch closed,} \\ \boldsymbol{0} & \text{switch open.} \end{cases}$$

Symbols: $\boldsymbol{x}$=position, $\boldsymbol{x}'$=velocity, $\boldsymbol{\omega}$=angle of rotation, $\boldsymbol{\tau}$=stress, $\boldsymbol{i}$=moment of inertia, $\boldsymbol{e}$=electromotive force, $\boldsymbol{F}$=force.

Constants: $\boldsymbol{k}$=Hooke's constant, $\boldsymbol{r}$=radius. Constants $\boldsymbol{n}$, $\boldsymbol{K}$ are from ME conventions.

Revised Differential Equation Model

Newtonian mechanics:

$$\boldsymbol{m}_{\mathsf{cage}}\boldsymbol{x}''_{\mathsf{cage}} = \boldsymbol{k}_{\mathsf{cable}}(\boldsymbol{x}_{\mathsf{rim}} - \boldsymbol{x}_{\mathsf{cage}}) - \boldsymbol{m}_{\mathsf{cage}}\boldsymbol{g}$$

Drum:

$$\begin{aligned} \boldsymbol{x}'_{\mathsf{rim}} &= \boldsymbol{r}_{\mathsf{drum}}\boldsymbol{\omega}_{\mathsf{drum}} \\ \boldsymbol{\omega}_{\mathsf{drum}} &= \boldsymbol{r}_{\mathsf{drum}}\boldsymbol{F}_{\mathsf{cable}} \end{aligned}$$

Motor:

$$\begin{aligned} \boldsymbol{\omega}_{\mathsf{motor}} &= \boldsymbol{e}_{\mathsf{motor}}/\boldsymbol{K}_{\mathsf{motor}} \\ \boldsymbol{i}_{\mathsf{motor}} &= \boldsymbol{\tau}_{\mathsf{motor}}/\boldsymbol{K}_{\mathsf{motor}} \end{aligned}$$

Gear Train:

$$\begin{aligned} \boldsymbol{\omega}_{\mathsf{drum}} &= \boldsymbol{n}_{\mathsf{gear}}\boldsymbol{\omega}_{\mathsf{motor}} \\ \boldsymbol{\tau}_{\mathsf{motor}} &= \boldsymbol{n}_{\mathsf{gear}}\boldsymbol{\tau}_{\mathsf{drum}} \end{aligned}$$

Switch with RC-circuit and with electrical-mechanical power analysis:

$$\boldsymbol{C}\boldsymbol{e}'_{\mathsf{motor}} = \frac{\boldsymbol{e}_{\mathsf{switch}} - \boldsymbol{e}_{\mathsf{motor}}}{\boldsymbol{R}} - \frac{\boldsymbol{r}_{\mathsf{drum}}\boldsymbol{n}_{\mathsf{gear}}}{\boldsymbol{K}_{\mathsf{motor}}}\boldsymbol{k}_{\mathsf{cable}}\left(\boldsymbol{x}_{\mathsf{rim}} - \boldsymbol{x}_{\mathsf{cage}}\right)$$

Sample Constants

$$
\begin{aligned}
\boldsymbol{R} &= 10 \text{ ohms} \\
\boldsymbol{C} &= 0.1 \text{ farads} \\
\boldsymbol{K}_{\mathsf{motor}} &= 0.03 \text{ Newton-meters/amp} \\
\boldsymbol{n}_{\mathsf{gear}} &= 0.02 \\
\boldsymbol{r}_{\mathsf{drum}} &= 0.05 \text{ meters} \\
\boldsymbol{k}_{\mathsf{cable}} &= 200000 \text{ Newton/meter} \\
\boldsymbol{m}_{\mathsf{cage}} &= 200 \text{ kilograms}
\end{aligned}
$$

Differential Equation Variables

$\boldsymbol{t}$=time	
$\boldsymbol{x_1} = \boldsymbol{x}_{\mathsf{cage}}(\boldsymbol{t})$	cage position
$\boldsymbol{x_2} = \boldsymbol{x}'_{\mathsf{cage}}(\boldsymbol{t})$	cage velocity
$\boldsymbol{x_3} = \boldsymbol{x}_{\mathsf{rim}}(\boldsymbol{t})$	rim position

Model Analysis

There are two methods for analysis of the differential equations model.

- Standard linear system solution methods using an equivalent matrix differential system $\boldsymbol{x}'' = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{F}(\boldsymbol{t})$.
- Laplace analysis, in particular, a transfer function analysis. This is the preferred method for most engineering fields.

The findings, which made the designer happy:

- The time constant which was initially 1 second increased to 3 seconds with the RC-circuit.
- The frequency of oscillation increased from about 5 Hertz to 9 Hertz.
- The amplitude of the oscillation decreased.