Differential Equations 2280
Midterm Exam 3
Wednesday, 22 April 2009

Instructions: This in-class exam is 50 minutes. No calculators, notes, tables or books. No answer check is expected. Details count 75%. The answer counts 25%. Each problem is scored 100.

Please discard this sheet after reading it.

Use this page to start your solution. Attach extra pages as needed, then staple.

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1. (ch7) Arrowsmith [Page]

(a) [25%] Display the integral formula for the direct Laplace Transform of $\dfrac{t}{1+t}$. Explain why the integral exists, citing theorems.

(b) [25%] Derive the formula $\mathcal{L}(f'(t)) = s\mathcal{L}(f(t)) - f(0)$.

(c) [50%] Solve $x''' + 3x'' + 2x' = 0$, $x(0) = x'(0) = 0$, $x''(0) = 1$ by Laplace's Method.

(d) [50%] Solve the system $x' = x + y$, $y' = x$, $x(0) = 1$, $y(0) = 0$ by Laplace's Method.

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2. (ch5) Cummings [Beattie]

The eigenanalysis method says that the system $\mathbf{x}' = A\mathbf{x}$ has general solution $\mathbf{x}(t) = c_1\mathbf{v}_1 e^{\lambda_1 t} + c_2\mathbf{v}_2 e^{\lambda_2 t} + c_3\mathbf{v}_3 e^{\lambda_3 t}$. In the solution formula, $(\lambda_i, \mathbf{v}_i)$, $i = 1, 2, 3$, is an eigenpair of A. Given

$$A = \begin{bmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 0 & 0 & 7 \end{bmatrix},$$

then

(a) [75%] Display eigenanalysis details for A.

(b) [25%] Display the solution $\mathbf{x}(t)$ of $\mathbf{x}'(t) = A\mathbf{x}(t)$.

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3. (ch5) Karrens [Harris]

(a) [50%] The eigenvalues are 2, 3, 4, 5 for the matrix $A = \begin{bmatrix} 4 & 1 & -1 & 0 \\ 1 & 4 & -2 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 2 & 4 \end{bmatrix}$.

Display the general solution of $\mathbf{u}' = A\mathbf{u}$ according to Putzer's spectral formula. Don't expand matrix products, in order to save time. However, do compute the coefficient functions r_1, r_2, r_3. Given below is the answer for r_4, to shorten the computation.

$$r_4(t) = -\frac{1}{6}e^{2t} + \frac{1}{2}e^{3t} - \frac{1}{2}e^{4t} + \frac{1}{6}e^{5t}.$$

(b) [50%] Using the same matrix A from part (a), display the solution of $\mathbf{u}' = A\mathbf{u}$ according to the Cayley-Hamilton Method. To save time, write out the system to be solved for the four vectors, and then stop, without solving for the vectors.

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4. (ch5) Wyatt [Williams]

(a) [50%] Display the solution of $\mathbf{u}' = \begin{pmatrix} 3 & 4 \\ 0 & 1 \end{pmatrix} \mathbf{u}$ according to the Laplace Resolvent Method. To save time, do not evaluate the constants in partial fractions.

(b) [50%] Display the solution of $\mathbf{u}' = \begin{pmatrix} 3 & 4 \\ 0 & 1 \end{pmatrix} \mathbf{u}$ according to the Eigenanalysis Method.

(c) [50%] Display the exponential matrix e^{At} for the system $\mathbf{u}' = \begin{pmatrix} 3 & 4 \\ 0 & 1 \end{pmatrix} \mathbf{u}$.

(d) [50%] Display the variation of parameters formula for the system below, but do not do any integrations, in order to save time.

$$\mathbf{u}' = \begin{pmatrix} 3 & 4 \\ 0 & 1 \end{pmatrix} \mathbf{u} + \begin{pmatrix} e^{-t} \\ 0 \end{pmatrix}.$$

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5. **(ch6)** Bennett [Osborne]

(a) [30%] Define *stable equilibrium* for $\mathbf{u}' = \mathbf{f}(\mathbf{u})$, a nonlinear 2-dimensional system in which $\mathbf{f}$ is continuously differentiable.

(b) [40%] Give an example of a linear 2-dimensional system with a stable spiral at equilibrium point $x = y = 0$. Draw a representative phase diagram about $x = y = 0$.

(c) [40%] Give an example of a linear 2-dimensional system with a stable center at equilibrium point $x = y = 0$. Draw a representative phase diagram about $x = y = 0$.

(d) [40%] Give an example of a linear 2-dimensional system with an unstable saddle at equilibrium point $x = y = 0$. Draw a representative phase diagram about $x = y = 0$.

(e) [30%] Assume a 2-dimensional predator-prey system $\mathbf{u}' = \mathbf{f}(\mathbf{u})$ has equilibrium points $(0, 0)$, $(160, 0)$, $(0, 180)$ and $(100, 90)$. Explain the possible physical meanings of the equilibria, e.g., extinction, explosion, carrying capacity.

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