Sample Midterm 2, Problem 2, part a

a) \( y^{(4)} + y^{(2)} = x + 2x^2 + x^3 + e^{-x} + x \sin x \)

\( f(x) \) use \( x^6 f(x) \) to generate possible atoms

for base atom: 1

\[
\frac{1, x, x^2, x^3}{x^4, x^5, x^6, x^7, x^8, x^9}
\]

duplicates from homogeneous solution

cross out from end to eliminate 6 total

for base atom \( e^{-x} \)

\[
\begin{array}{c}
\frac{e^{-x}}{x e^{-x}, x^2 e^{-x}, x^3 e^{-x}, x^4 e^{-x}, x^5 e^{-x}, x^6 e^{-x}}
\end{array}
\]

no duplicates from homogeneous sol.

cross out 6 total

for base atoms \( \sin x, \cos x \) (both treated the same)

\[
\frac{\sin x}{\cos x}
\]

duplicates from hom. sol.

cross out 6 total

\[
\frac{x \sin x, x^2 \sin x}{x \cos x, x^2 \cos x}
\]

cross out 6 total

\( Y_p = A x^4 + B x^5 + C x^4 + D x^9 + E e^{-x} + F x \sin x + G x^2 \sin x + H x \cos x + I x^2 \cos x \)
Sample Midterm 2, Problem 2, part b

b) \( y^{(4)} + y'' = t - \sin t \)

Transfer function \( T(s) \) is \( \mathcal{L}\{\text{LHS}\} \) with all initial conditions set to 0

\[
\mathcal{L}(y^{(4)} + y'') = \mathcal{L}(t - \sin t)
\]
\[
s^4L(y) + s^2L(y) = L(t - \sin t)
\]
\[
(s^4 + s^2)L(y) = \frac{1}{s^2} - \frac{1}{s^2 + 1}
\]
\[
L(y) = \frac{\frac{1}{s^2} - \frac{1}{s^2 + 1}}{s^4 + s^2}
\]
\[
= \frac{s^2 + 1 - s^2}{s^2(s^2 + 1)(s^2 + 1)} \left( \frac{1}{s^2(s^2 + 1)} \right)
\]
\[
= \frac{1}{s^4(s^2 + 1)^2}
\]

from partial fractions theory

without solving the partial fractions, it can be seen that the result will look like

\[
\mathcal{L}^{-1} \left( \frac{\frac{d_1}{s} + \frac{d_2}{s^2} + \frac{d_3}{s^3} + \frac{d_4}{s^4} + \frac{d_5 + d_6}{s^2 + 1}}{(s^2 + 1)^2} \right)
\]

these produce terms that are found in \( y_h \), so they are removed from the trial sol.

\[
y_h: \quad r^4 + r^2 = 0
\]
\[
r^2(r^2 + 1) = 0
\]
\[
y_h = C_1 + C_2 t + C_3 \cos t + C_4 \sin t
\]

The rest will produce:

\[
y_e = d_1 t^2 + d_2 t + d_3 t \cos t + d_4 t \sin t
\]