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Sample Midterm 2, Problem 2, part a

a) $y^{(6)} + y^{(4)} = \underbrace{x + 2x^2 + x^3 + e^{-x} + x\sin x}_{f(x)}$

y_h: $r^6 + r^4 = 0$

$r^4(r^2+1) = 0$

$y_h = c_1 + c_2 x + c_3 x^2 + c_4 x^3 + c_5 \sin x + c_6 \cos x$

use $x^6 f(x)$ to generate possible atoms

for base atom: 1

$\underbrace{1,\ x,\ x^2,\ x^3,}_{\text{duplicates from homogeneous solution}}\ x^4,\ x^5,\ x^6,\ x^7,\ \underbrace{x^8,\ x^9}_{\text{cross out from end to eliminate 6 total}}$

$(x^6(x + 2x^2 + x^3)) = x^7 + 2x^8 + x^9$

for base atom e^{-x}

$\underbrace{e^{-x},}_{\text{no duplicates from homogeneous sol.}}\ \underbrace{xe^{-x},\ x^2e^{-x},\ x^3e^{-x},\ x^4e^{-x},\ x^5e^{-x},\ x^6e^{-x}}_{\text{cross out 6 total}}$

$(x^6(e^{-x})) = x^6e^{-x}$

for base atoms $\sin x$, $\cos x$ (both treated the same)

$(x^6(x\sin x)) = x^7 \sin x$

$\underbrace{\begin{matrix}\sin x, \\ \cos x,\end{matrix}}_{\text{duplicates from homo. sol.}}\ \begin{matrix}x\sin x, & x^2\sin x \\ x\cos x, & x^2\cos x,\end{matrix}\ \underbrace{\begin{matrix}x^3\sin x, & x^4\sin x, & x^5\sin x, & x^6\sin x, & x^7\sin x \\ x^3\cos x, & x^4\cos x, & x^5\cos x, & x^6\cos x, & x^7\cos x\end{matrix}}_{\text{cross out 6 total}}$

$y_p = Ax^4 + Bx^5 + Cx^6 + Dx^7 + Ee^{-x} + Fx\sin x + Gx^2\sin x + Hx\cos x + Ix^2\cos x$

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Sample Midterm 2, Problem 2, part b

b) $y^{(4)} + y'' = t - \sin t$

Transfer function $T(s)$ is $\mathcal{L}\{LHS\}$ with all initial conditions set to 0

$$\mathcal{L}(y^{(4)} + y'') = \mathcal{L}(t - \sin t)$$

$$s^4\mathcal{L}(y) + s^2\mathcal{L}(y) = \mathcal{L}(t - \sin t)$$

$$(s^4 + s^2)\mathcal{L}(y) = \frac{1}{s^2} - \frac{1}{s^2+1}$$

$$\mathcal{L}(y) = \frac{\frac{1}{s^2} - \frac{1}{s^2+1}}{s^4 + s^2}$$

$$= \frac{s^2+1-s^2}{s^2(s^2+1)}\left(\frac{1}{s^2(s^2+1)}\right)$$

$$= \frac{1}{s^4(s^2+1)^2}$$

$$= \frac{d_1}{s} + \frac{d_2}{s^2} + \frac{d_3}{s^3} + \frac{d_4}{s^4} + \frac{d_5 s + d_6}{s^2+1} + \frac{d_7 s + d_8}{(s^2+1)^2}$$ from partial fractions theory

without solving the partial fractions it can be seen that the result will look like

$$\mathcal{L}^{-1}\left(\underline{\frac{d_1}{s} + \frac{d_2}{s^2}} + \frac{d_3}{s^3} + \frac{d_4}{s^4} + \underline{\frac{d_5 s + d_6}{s^2+1}} + \frac{d_7 s + d_8}{(s^2+1)^2}\right)$$

these produce terms that are found in y_h, so they are removed from the trial sol.

y_h: $r^4 + r^2 = 0$

$r^2(r^2+1) = 0$

$y_h = c_1 + c_2 t + c_3\cos t + c_4 \sin t$

The rest will produce:

$$y_p = d_1 t^2 + d_2 t^3 + d_3 t\cos t + d_4 t \sin t$$