Differential Equations 2280
Sample Midterm Exam 2
Thursday, 25 March 2008

Instructions: This in-class exam is 50 minutes. No calculators, notes, tables or books. No answer check is expected. Details count 75%. The answer counts 25%.

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1. (ch3)

Using Euler's theorem on atoms and the characteristic equation for higher order constant-coefficient differential equations, write out the general solutions for (a), (b), (c).

(a) [25%] $y'' + 4y' + 4y = 0$

(b) [25%] $y^{(5)} + 4y^{(4)} = 0$

(c) [25%] Characteristic equation $r(r-3)(r^3 - 9r)^2(r^2+4)^3 = 0$

(d) [25%] Given $6x''(t) + 7x'(t) + 2x(t) = 0$, which represents an unforced damped spring-mass system with $m = 6$, $c = 7$, $k = 2$. Solve the differential equation [15%]. Classify the answer as over-damped, critically damped or under-damped [5%]. Illustrate in a physical model drawing the meaning of constants m, c, k [5%].

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2. (ch3)

(a) [60%] Determine for $y^{(6)} + y^{(4)} = x + 2x^2 + x^3 + e^{-x} + x \sin x$ the corrected trial solution for y_p according to the method of undetermined coefficients. Do not evaluate the undetermined coefficients! The trial solution should be the one with fewest atoms.

(b) [40%] The corrected trial solution in undetermined coefficients for $y^{(4)}(t) + y''(t) = t - \sin t$ can be constructed from $T(s)\mathcal{L}(t - s \sin t)$ where $T(s)$ is the transfer function. Show the Laplace and partial fraction details necessary to obtain the corrected trial solution.

Note 1. Don't find $y_p(t)$ – find the trial solution with symbols d_1, d_2, $\ldots$.

Note 2. Certain partial fraction terms are removed before forming the trial solution. Document where it happens, but don't explain why.

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3. (ch3 and ch7)

(a) [50%] Find by any applicable method the steady-state periodic solution for the equation $x'' + 4x' + 6x = 10\cos(2t)$.

(b) [50%] Find by variation of parameters a particular solution y_p for the equation $y'' + 3y' + 2y = xe^{2x}$.

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4. (ch7)

(a) [25%] Solve by Laplace's method $x'' + x = \cos t$, $x(0) = x'(0) = 0$.

(b) [10%] Does there exist $f(t)$ of exponential order such that $\mathcal{L}(f(t)) = \dfrac{s}{s+1}$? Details required.

(c) [15%] Linearity $\mathcal{L}(c_1 f + c_2 g) = c_1\mathcal{L}(f) + c_2\mathcal{L}(g)$ is one Laplace rule. State four other Laplace rules. Forward and backward table entries are not rules, which means $\mathcal{L}(1) = 1/s$ doesn't count.

(d) [25%] Solve by Laplace's resolvent method

$$\begin{aligned} x'(t) &= x(t) + y(t), \\ y'(t) &= 2x(t), \end{aligned}$$

with initial conditions $x(0) = -1$, $y(0) = 2$.

(e) [25%] Derive $y(t) = \int_0^t \sin(t-u) f(u) du$ by Laplace transform methods from the forced oscillator problem

$$y''(t) + y(t) = f(t), \quad y(0) = y'(0) = 0.$$

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5. (ch7)

(a) [25%] Solve $\mathcal{L}(f(t)) = \dfrac{10}{(s^2+8)(s^2+4)}$ for $f(t)$.

(b) [25%] Solve for $f(t)$ in the equation $\mathcal{L}(f(t)) = \dfrac{s+1}{s^2(s+2)}$.

(c) [20%] Solve for $f(t)$ in the equation $\mathcal{L}(f(t)) = \dfrac{s-1}{s^2+2s+5}$.

(d) [10%] Solve for $f(t)$ in the relation

$$\mathcal{L}(f) = \frac{d}{ds}\mathcal{L}(t^2 \sin 3t)$$

(e) [10%] Solve for $f(t)$ in the relation

$$\mathcal{L}(f) = \left(\mathcal{L}\left(t^3 e^{9t} \cos 8t\right)\right)\Big|_{s \to s+3}.$$

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