**Math 2280 Extra Credit Problems**  
Chapter 7  
S2009

**Due date:** Submit these problems when the last section of chapter 4 is due. Records are locked on that date and only corrected, never appended. The scores on Ch7 extra credit can replace any missing score for the entire semester.

**Submitted work.** Please submit one stapled package with this sheet on top. Kindly check-mark the problems submitted and label the problems [Extra Credit]. Label each problem with its corresponding problem number, e.g., [Xc7.3-20].

**Problem Xc7.3-20. (Inverse transform)**
Solve for \( f(t) \) in the relation \( \mathcal{L}(f(t)) = \frac{1}{s^4 - 8s^2 + 16} \). Use partial fractions in the details.

**Problem Xc7.3-24. (Inverse transform)**
Solve for \( f(t) \) in the relation \( \mathcal{L}(f(t)) = \frac{s}{s^4 + 4a^4} \), showing the details that give the answer \( f(t) = \frac{1}{2a^2} \sinh at \sin at \).

**Problem Xc7.4-12. (Inverse transform, convolution)**
Solve for \( f(t) \) in the relation \( \mathcal{L}(f(t)) = \frac{1}{s(s^2 + 4s + 5)} \). Instead of the convolution theorem, use partial fractions for the details. If you can see how, then check the answer with the convolution theorem.

**Problem Xc7.4-26. (Inverse transform techniques)**
Use the \( s \)-differentiation theorem in the details of solving for \( f(t) \) in the relation \( \mathcal{L}(f(t)) = \arctan \frac{3}{s + 2} \). You will need to apply the theorem \( \lim_{s \to \infty} \mathcal{L}(f(t)) = 0 \).

**Problem Xc7.4-40. (Series methods for transforms)**
Expand in a series, using Taylor series formulas, the function \( f(t) = \frac{\cos \sqrt{7}}{\sqrt{\pi}t} \). Then find \( \mathcal{L}(f(t)) \) as a series by Laplace transform of each series term, separately. Finally, re-constitute the series in variable \( s \) into elementary functions, namely \( e^{-1/s} \) divided by \( \sqrt{s} \).

**Problem Xc7.5-6. (Second shifting theorem, Heaviside step)**
Find the function \( f(t) \) in the relation \( \mathcal{L}(f(t)) = \frac{se^{-s}}{s^2 + \pi^2} \).

**Problem Xc7.5-14. (Transforms of piecewise functions)**
Let \( f(t) = \begin{cases} 
\cos \pi t & 0 \leq t \leq 2, \\
0 & t > 2.
\end{cases} \) Find \( \mathcal{L}(f(t)) \). Details should expand \( f(t) \) as a linear combination of Heaviside step functions.

**Problem Xc7.5-26. (Sawtooth wave)**
Let \( f(t + a) = f(t) \) and \( f(t) = t \) on \( 0 \leq t \leq a \). Then \( f \) is \( a \)-periodic and has a Laplace transform obtained from the periodic function formula. Show the details in the derivation to obtain the answer \( \mathcal{L}(f(t)) = \frac{1}{as^2} - \frac{e^{-as}}{s(1 - e^{-as})} \).

**Problem Xc7.5-28. (Modified sawtooth wave)**
Let \( f(t + 2a) = f(t) \) and \( f(t) = t \) on \( 0 \leq t \leq a \), \( f(t) = 0 \) on \( a < t \leq 2a \). Then \( f \) is \( 2a \)-periodic and has a Laplace transform obtained from the periodic function formula. Derive a formula for \( \mathcal{L}(f(t)) \).

**Problem Xc7.6-8. (Impulsive DE)**
Solve by Laplace methods $x'' + 2x' + x = \delta(t) - 2\delta(t - 1)$, $x(0) = 1$, $x'(0) = 1$.

Problem Xc7.6-18. (Switching circuit)
A passive LC-circuit has battery 6 volts and model equation $i'' + 100i = 6\delta(t) - 6\delta(t - 1)$, $x(0) = 1$, $x'(0) = 1$. The switch is closed at time $t = 0$ and opened again at $t = 1$. Solve the equation by Laplace methods and report the number of full cycles observed before the steady-state $i = 0$ is reached.

End of extra credit problems chapter 7.