**Name** ____________________ Class Time _____

**Math 2280 Extra Credit Problems**
**Chapter 7**
**S2009**

**Due date**: Submit these problems when the last section of chapter 4 is due. Records are locked on that date and only corrected, never appended. The scores on Ch7 extra credit can replace *any* missing score for the entire semester.

**Submitted work**. Please submit one stapled package with this sheet on top. Kindly check-mark the problems submitted and label the problems $\boxed{\textbf{Extra Credit}}$. Label each problem with its corresponding problem number, e.g., $\boxed{\text{Xc7.3-20}}$.

## Problem Xc7.3-20. **(Inverse transform)**

Solve for $f(t)$ in the relation $\mathcal{L}(f(t)) = \dfrac{1}{s^4 - 8s^2 + 16}$. Use partial fractions in the details.

## Problem Xc7.3-24. **(Inverse transform)**

Solve for $f(t)$ in the relation $\mathcal{L}(f(t)) = \dfrac{s}{s^4 + 4a^4}$, showing the details that give the answer $f(t) = \dfrac{1}{2a^2}\sinh at \sin at$

## Problem Xc7.4-12. **(Inverse transform, convolution)**

Solve for $f(t))$ in the relation $\mathcal{L}(f(t)) = \dfrac{1}{s(s^2 + 4s + 5)}$. Instead of the convolution theorem, use partial fractions for the details. If you can see how, then check the answer with the convolution theorem.

## Problem Xc7.4-26. **(Inverse transform techniques)**

Use the $s$-differentiation theorem in the details of solving for $f(t)$ in the relation $\mathcal{L}(f(t)) = \arctan\dfrac{3}{s+2}$. You will need to apply the theorem $\lim_{s\to\infty}\mathcal{L}(f(t)) = 0$.

## Problem Xc7.4-40. **(Series methods for transforms)**

Expand in a series, using Taylor series formulas, the function $f(t) = \dfrac{\cos 2\sqrt{t}}{\sqrt{\pi t}}$. Then find $\mathcal{L}(f(t))$ as a series by Laplace transform of each series term, separately. Finally, re-constitute the series in variable $s$ into elementary functions, namely $e^{-1/s}$ divided by $\sqrt{s}$.

## Problem Xc7.5-6. **(Second shifting theorem, Heaviside step)**

Find the function $f(t)$ in the relation $\mathcal{L}(f(t)) = \dfrac{se^{-s}}{s^2 + \pi^2}$.

## Problem Xc7.5-14. **(Transforms of piecewise functions)**

Let $f(t) = \begin{cases} \cos \pi t & 0 \le t \le 2, \\ 0 & t > 2. \end{cases}$ Find $\mathcal{L}(f(t))$. Details should expand $f(t)$ as a linear combination of Heaviside step functions.

## Problem Xc7.5-26. **(Sawtooth wave)**

Let $f(t + a) = f(t)$ and $f(t) = t$ on $0 \le t \le a$. Then $f$ is $a$-periodic and has a Laplace transform obtained from the periodic function formula. Show the details in the derivation to obtain the answer $\mathcal{L}(f(t)) = \dfrac{1}{as^2} - \dfrac{e^{-as}}{s(1 - e^{-as})}$.

## Problem Xc7.5-28. **(Modified sawtooth wave)**

Let $f(t + 2a) = f(t)$ and $f(t) = t$ on $0 \le t \le a$, $f(t) = 0$ on $a < t \le 2a$. Then $f$ is $2a$-periodic and has a Laplace transform obtained from the periodic function formula. Derive a formula for $\mathcal{L}(f(t))$.

## Problem Xc7.6-8. **(Impulsive DE)**

Solve by Laplace methods $x'' + 2x' + x = \delta(t) - 2\delta(t-1)$, $x(0) = 1$, $x'(0) = 1$.

### Problem Xc7.6-18. **(Switching circuit)**

A passive LC-circuit has battery 6 volts and model equation $i'' + 100i = 6\delta(t) - 6\delta(t-1)$, $x(0) = 1$, $x'(0) = 1$. The switch is closed at time $t = 0$ and opened again at $t = 1$. Solve the equation by Laplace methods and report the number of full cycles observed before the steady-state $i = 0$ is reached.

**End of extra credit problems chapter 7.**