6. (Linear differential equations) Version 2 for 2 April 2009: Do all three parts.
   (a) [30%] Solve for the general solution of $4y'' + 4y' + 25y = 0$.
   (b) [40%] The characteristic equation is $r^2(r+2)^2(r^2 - 4r + 5) = 0$. Find the general solution $y$ of the homogeneous constant-coefficient differential equation.
   (c) [30%] Find a homogeneous second order differential equation with constant coefficients which has two solutions $e^x$ and $e^{-x/5}$.

(a) $4r^2 + 4r + 25 = 0 \quad (2r+1)^2 + 24 = 0 \quad 2r+1 = \pm 2\sqrt{6}i \quad r = -\frac{1}{2} \pm \sqrt{6}i \quad \text{atoms} = e^{-x/2} \cos(\sqrt{6}x) \quad e^{-x/2} \sin(\sqrt{6}x) \quad y = 1. c. y \ 1 \ \text{atoms}$

(b) $4r^4 (r^2 - 4r + 5) = 0 \quad \text{roots} = 0, 0, 0, 0, 2 \pm i \quad \text{atoms} = 1, x, x^2, x^3, e^x \cos x, e^x \sin x \quad y = 1. c. \ 1 \ \text{atoms}$

(c) Roots = 1, −1/5
    Factors = $r-1, r+1/5$

\[
\begin{align*}
(r-1)(5r+1) &= 0 \\
5r^2 - 4r - 1 &= 0 \\
5y'' - 4y' - y &= 0
\end{align*}
\]