Name KEY

6. **(Linear differential equations)** Version 2 for 2 April 2009: Do all three parts.

(a) [30%] Solve for the general solution of $4y'' + 4y' + 25y = 0$.

(b) [40%] The characteristic equation is $r^2(r^+2r)^2(r^2 - 4r + 5) = 0$. Find the general solution y of the homogeneous constant-coefficient differential equation.

(c) [30%] Find a homogeneous second order differential equation with constant coefficients which has two solutions e^x and $e^x + e^{-x/5}$.

(a) $4r^2 + 4r + 25 = 0$

$(2r+1)^2 + 24 = 0$

$2r+1 = \pm 2\sqrt{6}\,i$

$r = -\frac{1}{2} \pm \sqrt{6}\,i$

atoms $= e^{-x/2}\cos(\sqrt{6}x)$, $e^{-x/2}\sin(\sqrt{6}x)$

$y =$ l.c. of atoms

(b) $4r^4(r^2-4r+5) = 0$

roots $= 0, 0, 0, 0, 2 \pm i$

atoms $= 1, x, x^2, x^3$, $e^{2x}\cos x$, $e^{2x}\sin x$

$y =$ l.c. of atoms

(c) roots $= 1, -1/5$

Factors $= r-1,\ r + 1/5$

$(r-1)(5r+1) = 0$

$5r^2 - 4r - 1 = 0$

$\boxed{5y'' - 4y' - y = 0}$

Use this page to start your solution. Attach extra pages as needed, then staple.