Name KEY

6. **(Linear differential equations)** Version 1 for 2 April 2009: Do all three parts.

(a) [30%] Solve for the general solution of $4y'' + 4y' + 73y = 0$.

(b) [40%] The characteristic equation is $r^2(r^2 - 2r)^2(r^2 - 2r + 10) = 0$. Find the general solution y of the homogeneous constant-coefficient differential equation.

(c) [30%] Find a homogeneous second order differential equation with constant coefficients which has two solutions $e^{-x/3}$ and $e^x + e^{-x/3}$.

(a) $4r^2 + 4r + 73 = 0$

$(2r+1)^2 + 72 = 0$

$2r+1 = \pm\sqrt{72}\, i$

$2r+1 = \pm 6\sqrt{2}\, i$

$r = -\frac{1}{2} \pm 3\sqrt{2}\, i$

atoms = $e^{-x/2}\cos(3\sqrt{2}x)$, $e^{-x/2}\sin(3\sqrt{2}x)$

y = l.c. of atoms

(b) $r^4(r-2)^2((r-1)^2 + 9) = 0$

roots = $0, 0, 0, 0, 2, 2, 1 \pm 3i$

atoms = $1, x, x^2, x^3, e^{2x}, xe^{2x}, e^x\cos(3x), e^x\sin(3x)$

y = l.c. of atoms

(c) roots are $-1/3, 1$

Factors are $r + 1/3$, $r - 1$

$(3r+1)(r-1) = 0$

$3r^2 + r - 3r - 1 = 0$

$3r^2 - 2r - 1 = 0$

$$\boxed{3y'' - 2y' - y = 0}$$

Use this page to start your solution. Attach extra pages as needed, then staple.