Math 2250 Extra Credit Problems
Chapter 9
January 2009

**Due date**: See the internet due date for 9.4, which is the due date for these problems. Records are locked on that date and only corrected, never appended.

**Submitted work**. Please submit one stapled package per problem. Kindly label problems [Extra Credit]. Label each problem with its corresponding problem number, e.g., Xc9.1-4. You may attach this printed sheet to simplify your work.

**Problem Xc9.1-4. (Phase Portraits)**
Find the equilibrium points for the system. Plot a phase diagram using the *maple* code below.

\[
\begin{align*}
\frac{dx}{dt} &= x - 2y + 3, \\
\frac{dy}{dt} &= x - y + 2.
\end{align*}
\]

**Example**: Plot the phase diagram of \(u' = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix} u + \begin{pmatrix} 4 \\ 5 \end{pmatrix}\) using *maple*.

```
with(DEtools):
equilEQ:=[0=x+2*y+4,0=3*y+5];
solve(equilEQ,{x,y});# find diagram center (a,b)
a:=-2/3;b:=-5/3;
de:=[diff(x(t),t)=x(t)+2*y(t)+4,diff(y(t),t)=3*y(t)+5];
ic:=[[x(0)=0,y(0)=-1],[x(0)=-1,y(0)=-1.5],[x(0)=0.5,y(0)=-2],
    [x(0)=0.5,y(0)=-1.5],[x(0)=-0.7,y(0)=-1.7]];
DEplot(de,[x(t),y(t)],t=-10..10,ic,x=a-2..a+2,y=b-2..b+2,stepsize=0.05);
```

**Problem Xc9.1-8. (Equilibrium Points)**
Find the equilibrium points for the system. Plot a phase diagram. The graph window should include the three equilibrium points.

\[
\begin{align*}
\frac{dx}{dt} &= x - 2y, \\
\frac{dy}{dt} &= 4x - x^3.
\end{align*}
\]

**Problem Xc9.1-18. (Stability)**
Determine if the equilibrium point \((0,0)\) is stable, asymptotically stable, or unstable. Identify the equilibrium point as a node, saddle, center or spiral by examination of its computer-generated direction field.

(a) \(x' = y, \ y' = -x\)

(b) \(x' = y, \ y' = -5x - 4y\)

(c) \(x = -2x, \ y' = -2y\)

(d) \(x = y, \ y' = x\)

**Problem Xc9.2-2. (Classification by Eigenvalues)**
Compute the eigenvalues of \(A\). Determine stability of equilibrium \((0,0)\) and classify as node (proper/improper), saddle, spiral, center.
(a) \( A \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \)
(b) \( A \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \)
(c) \( A \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix} \)
(c) \( A \begin{pmatrix} 1 & -2 \\ 2 & -3 \end{pmatrix} \)

Problem Xc9.2-12. (Phase Portrait)
Find the equilibrium point (it is unique) and plot by computer a phase diagram.
\[
\begin{align*}
\frac{dx}{dt} &= x + y - 7, \\
\frac{dy}{dt} &= 3x - y - 5.
\end{align*}
\]

Problem Xc9.2-22. (Almost Linear System)
Linearize the system at its equilibria and determine the stability and type of each. Plot a phase diagram by computer to verify the claims made.
\[
\begin{align*}
\frac{dx}{dt} &= 2x - 5y + x^3, \\
\frac{dy}{dt} &= 4x - 6y + y^4.
\end{align*}
\]

Problem Xc9.3-8. (Predator-Prey System)
Linearize the system at equilibrium point \((0,0)\). Verify that the phase diagram of the nonlinear system at \((0,0)\) is a saddle.
\[
\begin{align*}
\frac{dx}{dt} &= x(5 - x - y), \\
\frac{dy}{dt} &= y(-2 + x).
\end{align*}
\]

Problem Xc9.3-9. (Predator-Prey System)
Linearize the system at equilibrium point \((5,0)\). Verify that the phase diagram of the nonlinear system at \((5,0)\) is a saddle.
\[
\begin{align*}
\frac{dx}{dt} &= x(5 - x - y), \\
\frac{dy}{dt} &= y(-2 + x).
\end{align*}
\]

Problem Xc9.3-10. (Predator-Prey System)
Linearize the system at equilibrium point \((2,3)\). Verify that the phase diagram of the nonlinear system at \((2,3)\) is an asymptotically stable spiral.
\[
\begin{align*}
\frac{dx}{dt} &= x(5 - x - y), \\
\frac{dy}{dt} &= y(-2 + x).
\end{align*}
\]

Problem Xc9.4-4. (Almost Linear System)
Linearize at \((0, 0)\) and classify the equilibrium point \((0, 0)\) of the nonlinear system, using a phase diagram to verify the conclusion.

\[
\begin{align*}
\frac{dx}{dt} &= 2\sin x + \sin y, \\
\frac{dy}{dt} &= \sin x + 2\sin y.
\end{align*}
\]

Problem Xc9.4-8. (Almost Linear System)
Linearize at all equilibria and classify the equilibrium points of the nonlinear system. Use a phase diagram to verify the conclusions.

\[
\begin{align*}
\frac{dx}{dt} &= y, \\
\frac{dy}{dt} &= \sin \pi x - y.
\end{align*}
\]

End of extra credit problems chapter 9.