Project 8. Solve problems L8.1 to L8.5. The problem headers:

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_______ PROBLEM L8.1. EARTHQUAKE MODEL FOR A BUILDING.
_______ PROBLEM L8.2. TABLE OF NATURAL FREQUENCIES AND PERIODS.
_______ PROBLEM L8.3. UNDETERMINED COEFFICIENTS STEADY-STATE SOL
_______ PROBLEM L8.4. PRACTICAL RESONANCE.
_______ PROBLEM L8.5. EARTHQUAKE DAMAGE.

SIX FLOOR Model.
Refer to the textbook of Edwards-Penney, section 7.4, page 437.
Consider a building with six floors each weighing 50 tons. Each floor corresponds to a restoring Hooke’s force with constant k=5 tons/foot. Assume that ground vibrations from the earthquake are modeled by \((1/4)\cos(wt)\) with period \(T=2\pi/w\).

**PROBLEM L8.1. BUILDING MODEL FOR AN EARTHQUAKE.**
Model the 6-floor problem in Maple.
Define the 6 by 6 mass matrix \(M\) and Hooke’s matrix \(K\) for this system and convert \(Mx''=Kx\) into the system \(x''=Ax\) where \(A\) is defined by textbook equation (1), page 437.

Sanity check: Mass \(m=3125\), and the 6x6 matrix contains fraction 16/5.
Then find the eigenvalues of the matrix \(A\) to six digits, using the Maple command "eigenvals(A)."
Sanity check: All six eigenvalues should be negative.

# Sample Maple code for a model with 4 floors.
# Use maple help to learn about evalf and eigenvals.
# \(A:=\text{matrix}([[-20,10,0,0], [10,-20,10,0], [0,10,-20,10],[0,0,10,-10]]);\)
# with(linalg): evalf(eigenvals(A));

# Problem L8.1
# Define \(k\), \(m\) and the 6x6 matrix \(A\).
# with(linalg): evalf(eigenvals(A));

**PROBLEM L8.2. TABLE OF NATURAL FREQUENCIES AND PERIODS.**
Refer to figure 7.4.17, page 437.
Find the natural angular frequencies \(\omega=\sqrt{-\lambda}\) for the six story building and also the corresponding periods \(2\pi/\omega\), accurate to six digits. Display the answers in a table.
Compare with answers in Figure 7.4.17, page 437, for the 7-story case.

# Sample code for a 4x3 table, 4-story building.
# Use maple help to learn about nops and printf.
# Problem L8.2
# ev:=[fill this in]: n:=nops(ev):
# Omega:=lambda -> sqrt(-lambda): format:="%10.6f %10.6f %10.6f\n":
# seq(printf(format,ev[i],Omega(ev[i]),2*evalf(Pi)/Omega(ev[i])),i=1..n);

# Problem L8.3
# Define w, u, b, A, Au, c
# evalf(c[1],2);

# PROBLEM L8.4. PRACTICAL RESONANCE.
Consider the forced equation \( x' = Ax + \cos(\omega t)b \) of L8.3 above with 
\( b := 0.25 \cdot w \cdot w \cdot \text{vector}([1,1,1,1,1,1]) \). 
Practical resonance can occur if a component of \( x(t) \) has large 
amplitude compared to the vector norm of \( b \). For example, an earthquake 
might cause a small 3-inch excursion on level ground, but the 
building's floors might have 50-inch excursions, enough to destroy
Let \( \text{Max}(c) \) denote the maximum modulus of the components of vector \( c \).

Plot \( g(T) = \text{Max}(c(w)) \) with \( w = \frac{(2\pi)}{T} \) for periods \( T = 0 \) to \( T = 6 \), ordinates \( \text{Max} = 0 \) to \( \text{Max} = 10 \), the vector \( c(w) \) being the answer produced in L8.3 above. Compare your figure to the textbook Figure 7.4.18, page 438.

# Sample maple code to define the function Max(c), 4-floor building.
# Use maple help to learn about norm, vector, subs and linsolve.

```
# with(linalg):
# w := 'w': Max := c -> norm(c, infinity); u := w*w:
# b := 0.25*w*w*vector([1, 1, 1, 1]):
# A := matrix([[-20, 10, 0, 0], [10, -20, 10, 0], [0, 10, -20, 10], [0, 0, 10, -10]]):
# Au := evalm(A + u*diag(1, 1, 1, 1));
# C := w*w -> subs(w=ww, linsolve(Au, -b)):
# plot(Max(C(2*Pi/r)), r=0..6, 0..10, numpoints=150);
```

PROBLEM L8.5. EARTHQUAKE DAMAGE.
The maximum amplitude plot of L8.4 can be used to detect the
of earthquake damage for a given
ground vibration of period \( T \). A ground vibration \((1/4)\cos(\omega t)\),
\( T = 2\pi/\omega \), will be assumed, as in L8.4.

(a) Re-plot the amplitudes in L8.4 for periods 1.5 to 5.5 and
amplitudes 5 to 10.
There will be five spikes.
(b) Create five zoom-in plots, one for each spike, choosing a
\( T \)-interval that shows the full spike.
(c) Determine from the five zoom-in plots approximate intervals for
the period \( T \) such that some floor in the building will undergo
excursions from equilibrium in excess of 5 feet.

# Example: Zoom-in on a spike for amplitudes 5 feet to 10 feet,
#periods 1.97 to 2.01.
#with(linalg): w := 'w': Max := c -> norm(c, infinity); u := w*w:
#Au := matrix([[-20+u, 10, 0, 0], [10, -20+u, 10, 0],
#[0, 10, -20+u, 10], [0, 0, 10, -10+u]]):
#b := 0.25*w*w*vector([1, 1, 1, 1]):
#C := w*w -> subs(w=ww, linsolve(Au, -b)):
#plot(Max(C(2*Pi/r)), r=1.97..2, 01, 5..10, numpoints=150);

# PROBLEM L8.5. WARNING: Save your file often!!
#(a) Re-plot the five spikes.
# plot(Max(C(2*Pi/r)), r=1.5..5.5, 5..10, numpoints=150);
#(b) Plot five zoom-in graphs.
# one:=1.79..1.83:plot(Max(C(2*Pi/r)),r=one,5..10,numpoints=150);
# two:=???:plot(Max(C(2*Pi/r)),r=two,5..10,numpoints=150);
# three:=???:plot(Max(C(2*Pi/r)),r=three,5..10,numpoints=150);
# four:=???:plot(Max(C(2*Pi/r)),r=four,5..10,numpoints=150);
# five:=???:plot(Max(C(2*Pi/r)),r=five,5..10,numpoints=150);
#(c) Print period ranges.
# PeriodRanges:=[one,two,three,four,five];