Name ____________________________ Class Time _____

Math 2250 atomRoot Extra Credit Problem
Chapter 5, section 5.5
March 2007

Due date: See the internet due date for 6.1, which is the due date for this problem. Records are locked on that date and only corrected, never appended.

Submitted work. Please submit one stapled package per problem. Kindly label problems $\boxed{\textbf{Extra Credit}}$. Label each problem with its corresponding problem number. You may attach this printed sheet to simplify your work.

Atoms. An **atom** is a term of the form $x^k e^{ax}$, $x^k e^{ax}\cos bx$ or $x^k e^{ax}\sin bx$. The symbol k is a non-negative integer. Symbols a and b are real numbers with $b > 0$. In particular, 1, x, x^2, e^x, $\cos x$, $\sin x$ are atoms. Any distinct list of atoms is linearly independent.

Roots and Atoms. Define **atomRoot**(A) as follows. Symbols α, β, r are real numbers, $\beta > 0$ and k is a non-negative integer.

atom A	**atomRoot**(A)
$x^k e^{rx}$	r
$x^k e^{\alpha x}\cos\beta x$	$\alpha + i\beta$
$x^k e^{\alpha x}\sin\beta x$	$\alpha + i\beta$

The fixup rule for undetermined coefficients can be stated as follows:

> *Compute* **atomRoot**(A) *for all atoms A in the trial solution. Assume r is a root of the characteristic equation of multiplicity k. Search the trial solution for atoms B with* **atomRoot**$(B) = r$*, and multiply each such B by x^k. Repeat for all roots of the characteristic equation.*

Problem Ex5.5-atomRoot. (Counts as 2 problems or 200)

1. Evaluate **atomRoot**(A) for $A = 1$, x, x^2, e^{-x}, $\cos 2x$, $\sin 3x$, $x\cos\pi x$, $e^{-x}\sin 3x$, x^3, e^{2x}, $\cos x/2$, $\sin 4x$, $x^2\cos x$, $e^{3x}\sin 2x$.

2. Let $A = xe^{-2x}$ and $B = x^2e^{-2x}$. Verify that **atomRoot**(A) = **atomRoot**(B).

3. Let $A = xe^{-2x}$ and $B = x^2e^{2x}$. Verify that **atomRoot**$(A) \neq$ **atomRoot**(B).

4. Atoms A and B are said to be **related** if and only if the derivative lists A, A', ... and B, B', ... share a common atom. Prove: atoms A and B are related if and only if **atomRoot**(A) = **atomRoot**(B).

End of extra credit problems chapter 5.