Name ________________________ **2270-2**

Introduction to Linear Algebra 2270-2
Revised Sample, Midterm Exam 2 Spring 2007
Exam Date: 28 March

Instructions. This exam is designed for 50 minutes. Calculators, books, notes and computers are not allowed.

1. (Matrices, determinants and independence) Do two parts.

(a) Prove that the pivot columns of A form a basis for $\mathbf{im}(A)$.

(b) Suppose A and B are both $n \times m$ of rank m and $\mathbf{rref}(A) = \mathbf{rref}(B)$. Prove or give a counterexample: the column spaces of A and B are identical.

Start your solution on this page. Please staple together any additional pages for this problem.

2. (Kernel and similarity) Do two parts.

(a) Illustrate the relation $\mathbf{rref}(A) = E_k \cdots E_2 E_1 A$ by a frame sequence and explicit elementary matrices for the matrix

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 1 & 0 \\ 2 & 2 & 0 \end{pmatrix}.$$

(b) Prove or disprove: $\mathbf{ker}(\mathbf{rref}(BA)) = \mathbf{ker}(A)$, for all invertible matrices B.

Start your solution on this page. Please staple together any additional pages for this problem.

3. (Independence and bases) Do two parts.

(a) Let A be a 12×15 matrix. Suppose that, for any possible independent set $\mathbf{v}_1, \ldots, \mathbf{v}_k$, the set $A\mathbf{v}_1, \ldots, A\mathbf{v}_k$ is independent. Prove or give a counterexample: $\mathbf{ker}(A) = \{\mathbf{0}\}$.

(b) Let V be the vector space of all polynomials $c_0 + c_1 x + c_2 x^2$ under function addition and scalar multiplication. Prove that $1 - x$, $2x$, $(x-1)^2$ form a basis of V.

Start your solution on this page. Please staple together any additional pages for this problem.

4. (Linear transformations) Do two parts.

(a) Let L be a line through the origin in $\mathcal{R}^3$ with unit direction $\mathbf{u}$. Let T be a reflection through L. Define T precisely. Display its representation matrix A, i.e., $T(\mathbf{x}) = A\mathbf{x}$.

(b) Let T be a linear transformation from $\mathcal{R}^n$ into $\mathcal{R}^m$. Let $\mathbf{v}_1, \ldots, \mathbf{v}_n$ be the columns of I and let A be the matrix whose columns are $T(\mathbf{v}_1), \ldots, T(\mathbf{v}_n)$. Prove that $T(\mathbf{x}) = A\mathbf{x}$.

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5. (Vector spaces)

(a) Show that the set of all 4×3 matrices A which have exactly one element equal to 1, and all other elements zero, form a basis for the vector space of all 4×3 matrices.

(b) Let $S = \{ \begin{pmatrix} a & b \\ -a & 2b \end{pmatrix} : a, b \text{ real}\}$. Find a basis for S.

(c) Let V be the vector space of all functions defined on the real line, using the usual definitions of function addition and scalar multiplication. Let S be the set of all polynomials of degree less than 5 (e.g., $x^4 \in V$ but $x^5 \notin V$) that have zero constant term. Prove that S is a subspace of V.

Start your solution on this page. Please staple together any additional pages for this problem.