Math 2270 Extra Credit Problems
Chapter 3
March 2007

Due date: See the internet due date for 5.5, which is the due date for these problems. Records are locked on that date and only corrected, never appended.

Submitted work. Please submit one stapled package per problem. Kindly label problems $\boxed{\textbf{Extra Credit}}$. Label each problem with its corresponding problem number. You may attach this printed sheet to simplify your work.

Problem Ex3.1-12. (**Image and kernel**)

For the matrix C below, display a frame sequence from C to $\mathbf{rref}(C)$. Write the image of C as the span of the pivot columns of C. Write the kernel of C as the list of partial derivatives $\partial \mathbf{x}/\partial c_1$, etc, where $\mathbf{x}$ is the vector general solution to $C\mathbf{x} = \mathbf{0}$.

$$C = \begin{pmatrix} 1 & -1 & -1 & 1 & 1 \\ -1 & 1 & 0 & 3 & 1 \\ 1 & -1 & -2 & 0 & 4 \\ 1 & -1 & -3 & 4 & 6 \end{pmatrix}$$

Problem Ex3.1-22. (**Geometry of a linear transformation**)

Give an example of a 3×3 matrix A such that $T(\mathbf{x}) = A\mathbf{x}$ has image equal to the plane through the three points $(0,0,0)$, $(0,1,1)$, $(1,1,0)$.

Problem Ex3.1-38. (**Image and kernel**)

Express the image of the matrix A as the kernel of a matrix B. The matrix B can be a different size than A.

$$A = \begin{pmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 0 & 3 \\ 1 & -1 & -2 & 0 \\ 1 & -1 & -3 & 4 \end{pmatrix}$$

Problem Ex3.1-50. (**Kernel and image**)

Let A be an $n \times n$ matrix and $B = \mathbf{rref}(A)$. Do A and B have the same kernels and images? Prove each assertion or give a counterexample.

Problem Ex3.2-18. (**Redundant vectors**)

Identify the redundant vectors in the list, by application of the pivot theorem.

$$\begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 12 \\ 0 \\ 4 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \\ 0 \\ 1 \end{pmatrix}.$$

Problem Ex3.2-22. (**Pivot columns**)

Express the non-pivot columns of A as linear combinations of the pivot columns of A.

$$A = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

Problem Ex3.2-46. (Basis for ker(A))

Find a basis for the kernel of A, using frame sequence methods.

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 2 & 3 \\ 0 & -1 & 2 & 3 \end{pmatrix}$$

Problem Ex3.2-48. (Independence with symbols)

Determine all values of the symbols a and b such that the following vectors are linearly independent.

$$\begin{pmatrix} a \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} a \\ 2b \\ 0 \\ 2b \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 3b \\ 0 \\ a-b \end{pmatrix}.$$

Problem Ex3.3-10. (Basis for ker(A) and image(A))

Find redundant columns by inspection and then find a basis for **image**(A) and **ker**(A).

$$A = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 2 & 2 & 2 \end{pmatrix}$$

Problem Ex3.3-24. (Basis for ker(A) and image(A))

Find **rref**(A) and then a basis for **ker**(A) and **image**(A).

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

Problem Ex3.3-52. (Row space basis)

Find a basis for the row space of A consisting of columns of A^T.

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 3 & 3 & 1 & 3 \\ 5 & 3 & 1 & 3 \end{pmatrix}$$

Problem Ex3.3-64. (Kernels of matrices)

Prove or disprove that **ker**(A) = **ker**(B).

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 3 & 3 & 1 & 3 & 0 \\ 5 & 3 & 1 & 3 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 3 & 3 & 1 & 3 & 0 \\ 5 & 3 & 1 & 3 & 0 \end{pmatrix}.$$

Problem Ex3.4-18. (Coordinates and spanning sets)

Let $V = \mathbf{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, where the vectors are displayed below. Test for $\mathbf{x}$ in V, and if true, then report the coordinates of $\mathbf{x}$ relative to the the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$.

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 1 \\ 3 \\ 6 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} 7 \\ 1 \\ 3 \end{pmatrix}.$$

Problem Ex3.4-30. (Matrix of a linear transformation)

Find the matrix B of the linear transformation T relative to the basis $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$.

$$T(\mathbf{x}) = \begin{pmatrix} 5 & -4 & -2 \\ -4 & 5 & -2 \\ -2 & -2 & 8 \end{pmatrix},$$

$$\mathbf{v}_1 = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}.$$

Problem Ex3.4-46. (Basis of a plane)

Consider the plane $2x_1 - 3x_2 + 4x_3 = 0$. Choose a basis $\mathbf{v}_1$, $\mathbf{v}_2$ for the plane, arbitrarily, your choice. Then determine the vector $\mathbf{x}$ which has coordinates 2, -1 relative to this basis.

End of extra credit problems chapter 3.