Name _____ Class Time _____

Math 2270 Extra Credit Problems
Chapter 1
January 2007

Due date: See the internet due date for 2.1, which is the due date for these problems. Records are locked on that date and only corrected, never appended.

Submitted work. Please submit one stapled package per problem. Kindly label problems **Extra Credit**. Label each problem with its corresponding problem number. You may attach this printed sheet to simplify your work.

Problem Ex1.1-26. (Three possibilities)

Determine which values of k correspond to (a) a unique solution, (b) no solution or (c) infinitely many solutions.

$$\left|\begin{array}{rcrcrcl} x & + & 2y & + & kz & = & 0 \\ 2x & + & 4y & + & (k+1)z & = & 2 \\ 3x & + & 6y & + & (2k+1)z & = & 2 \end{array}\right|$$

Problem Ex1.1-30. (Polynomial interpolation)

Find the polynomial $f(x) = a + bx + cx^2$ which passes through the points $(1, 9)$, $(2, 24)$, $(3, 47)$.

Problem Ex1.1-32. (Polynomial interpolation)

Find all polynomials $f(x) = a + bx + cx^2$ which pass through the points $(1, 10)$, $(2, 28)$ and $f'(3) = 33$.

Problem Ex1.2-22. (RREF)

Report five types of 3×4 matrices in RREF form.

Problem Ex1.2-28. (Combo rule)

Consider the following systems.

(1)
$$\left|\begin{array}{lcccl} a_{11}x_1 & + & \cdots & + & a_{1n}x_n = b_1 \\ & & \vdots & & \\ a_{m1}x_1 & + & \cdots & + & a_{mn}x_n = b_m \end{array}\right|$$

(2)
$$\left|\begin{array}{lcccl} a_{11}x_1 & + & \cdots & + & a_{1n}x_n = b_1 \\ & & \vdots & & \\ c_{k1}x_1 & + & \cdots & + & c_{kn}x_n = d_k \\ & & \vdots & & \\ a_{m1}x_1 & + & \cdots & + & a_{mn}x_n = b_m \end{array}\right|$$

We assume the systems identical except for equation k, which is obtained in the second system by applying a combination rule `combo(r,k,c)` to the first system. We assume $r \neq k$ and then the coefficients in the second system are given by

$$c_{kj} = a_{kj} + ca_{rj}, \quad j = 1, \ldots, n, \quad d_k = b_k + cb_r.$$

(a) Prove that every solution of system (1) is a solution of system (2).

(b) Prove that every solution of system (2) is a solution of system (1).

Problem Ex1.2-30. (Polynomial interpolation)

Find the polynomial $f(x) = a + bx + cx^2 + dx^3$ which satisfies $f(1) = 8$, $f(2) = 24$, $f(3) = 24$, $f(4) = 110$.

Problem Ex1.3-26. (Matrix algebra)

Find a 3×3 matrix A which satisfies the following relations.

$$A \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad A \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad A \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

End of extra credit problems chapter 1.