Math 2250
Report Contents 2.4,2.5,2.6 Problem 6
January 2007

Exact Solution 2.4,2.5,2.6-#6

The exact solution for $y' = -2xy$, $y(0) = 2$ should be derived in L3.1. In the numerical work, L3.2, L3.3, L3.4, this symbolic derivation is only referenced (do not derive again!). The answer:

$$y = 2e^{-x^2}.$$

A table of exact values is required in order to make comparison tables. Make this table for each problem separately, as the values used vary from one comparison to another.

2.4 Notes

Numerical Solution 2.4-#6

This work has to be done before you can write the report. Please write a report that references an appendix to be attached as a worksheet print; see below for the content of the appendix. Include here handwritten material that describes the Euler algorithm as applied to problem #6, then reference the worksheet for results.

The maple code referenced in the 19-page internet document *Numerical DE Manuscript* will be used. There is a text file of the actual code segments in the internet document document *Numerical DE maple coding hints*. Both are located at the course web site.

Sample Euler code:

```
# Warning: These snips of code made for y'=1-x-y, y(0)=3.
#          Code computes approx values for y(0.1) to y(0.5).
# 'Dots' is the list of dots for connect-the-dots graphics.
# =======================================
# Euler. Group 1, initialize.
 f:=(x,y)->1-x-y:
 x0:=0:y0:=3:h:=0.1:Dots:=[x0,y0]:
# Group 2, repeat 5 times. Euler's method
 for j from 1 to 5 do
 Y:=y0+h*f(x0,y0);
 x0:=x0+h:y0:=Y:Dots:=Dots,[x0,y0];
 end do:
# Group 3, show Dots, then plot.
 Dots[1],Dots[2],Dots[3],Dots[4],Dots[5],Dots[6];
 plot([Dots]);
```

To start, get the sample code to produce correct answers to the example supplied in the text file source. Once correct, modify the code for #6. The step size $h = 0.25$ produces a dot table of 3 rows, whereas the step size $h = 0.1$ makes a dot table with 6 rows.

Comparison Table 2.4-#6

The comparison will be 3 rows in 2.4-#6, which means half the $h = 0.1$ data is not used in the report. The table should list x, $y1$, $y2$, y where $y1$ is the $h = 0.25$ approximate value, $y2$ is the $h = 0.1$ approximate value and y is the exact value.

Graphics 2.4-#6

There should be three graphics, one for $h = 0.25$, one for $h = 0.1$ and one for the exact solution. All are produced in maple. Reference the maple worksheet appendix.

Appendix: Hand Solution Steps 2.4-#6

Include a derivation of the numerical values for $x = x_0 + h$, $x_0 = 0$, for each case $h = 0.1$ and $h = 0.05$. Show all steps by hand. This is the only cross-check on the numerics.

Appendix: Maple Worksheet 2.4-#6

Attach a print of the maple worksheet that contains all computer code and data used in 2.4-#6. Reference this appendix during the report.

2.5 Notes

Numerical Solution 2.5-#6

This work has to be done before you can write the report. Please write a report that references an appendix to be attached as a worksheet print; see below for the content of the appendix. Include here handwritten material that describes the Heun (modified Euler) algorithm as applied to problem #6, then reference the worksheet for results.

Sample Heun code:

```
# Warning: These snips of code made for y'=1-x-y, y(0)=3.
#          Code computes approx values for y(0.1) to y(0.5).
# 'Dots' is the list of dots for connect-the-dots graphics.
# ========================================
# Heun [=Modified Euler]. Group 1, initialize.
 f:=(x,y)->1-x-y:
 x0:=0:y0:=3:h:=0.1:Dots:=[x0,y0]:
# Group 2, repeat 5 times. Heun's method
 for j from 1 to 5 do
 Y1:=y0+h*f(x0,y0);
 Y:=y0+h*(f(x0,y0)+f(x0+h,Y1))/2:
 x0:=x0+h:y0:=Y:Dots:=Dots,[x0,y0];
 end do:
# Group 3, show Dots, then plot.
 Dots[1],Dots[2],Dots[3],Dots[4],Dots[5],Dots[6];
 plot([Dots]);
```

To start, get the sample Heun code to produce correct answers to the example supplied in the text file source. Once correct, modify the code to apply to 2.5-#6. The step size $h = 0.1$ produces a dot table of 6 rows.

Comparison Table 2.5-#6

The comparison will be 6 rows in 2.5-#6. The table should list x, $y1$, y where $y1$ is the $h = 0.1$ approximate value and y is the exact value.

Graphics 2.5-#6

There should be two graphics, one for $h = 0.1$ and one for the exact solution. All are produced in maple. Reference the maple worksheet appendix.

Appendix: Hand Solution Steps 2.5-#6

Include a derivation of the numerical values for $x = x_0 + h$, $x_0 = 0$, for the case $h = 0.1$. Show all steps by hand. This is the only cross-check on the numerics.

Appendix: Maple Worksheet 2.5-#6

Attach a print of the maple worksheet that contains all computer code and data used in 2.5-#6. Reference this appendix during the production of the report.

2.6 Notes

Numerical Solution 2.6-#6

This work has to be done before you can write the report. Please write a report that references an appendix to be attached as a worksheet print; see below for the content of the appendix. Include here handwritten material that describes the RK4 algorithm as applied to problem #6, then reference the worksheet for results.

Sample RK4 code:

```
# Warning: These snips of code made for y'=1-x-y, y(0)=3.
#          Code computes approx values for y(0.25) to y(0.5).
# 'Dots' is the list of dots for connect-the-dots graphics.
# =========================================
# RK4. Group 1, initialize.
 f:=(x,y)->1-x-y:
 x0:=0:y0:=3:h:=0.25:Dots:=[x0,y0]:
# Group 2, repeat one time. RK4 method
 for j from 1 to 1 do
 k1:=h*f(x0,y0):
 k2:=h*f(x0+h/2,y0+k1/2):
 k3:=h*f(x0+h/2,y0+k2/2):
 k4:=h*f(x0+h,y0+k3):
 Y:=y0+(k1+2*k2+2*k3+k4)/6:
 x0:=x0+h:y0:=Y:Dots:=Dots,[x0,y0];
 end do:
# Group 3, show Dots, then plot.
 Dots[1],Dots[2],Dots[3];
 plot([Dots]);
```

To start, get the sample RK4 maple code, referenced in the 19-page internet document *Numerical DE Manuscript*, to produce correct answers to the example supplied in the text file source. Once correct, modify the code for #6. The step size $h = 0.25$ produces a dot table of 3 rows.

Comparison Table 2.6-#6

The comparison will be 3 rows in 2.6-#6. The table should list x, $y1$, y where $y1$ is the $h = 0.25$ approximate value and y is the exact value.

Graphics 2.6-#6

There should be two graphics, one for $h = 0.25$ and one for the exact solution. All are produced in maple. Reference the maple worksheet appendix.

Appendix: Hand Solution Steps 2.6-#6

Skip this step for 2.6-#6, because the machine is likely more reliable than a hand calculation. Instead of a hand check, check the obtained answers against those already known for Euler and Heun methods.

Appendix: Maple Worksheet 2.6-#6

Attach a print of the maple worksheet that contains all computer code and data used in 2.6-#6. Reference this appendix during the production of the report.