

**Math 2250 Extra Credit Problems**  
**Chapter 6**  
**April 2007**

**Due date:** See the internet due date for 7.3, which is the due date for these problems. Records are locked on that date and only corrected, never appended.

**Submitted work.** Please submit one stapled package per problem. Kindly label problems Extra Credit. Label each problem with its corresponding problem number. You may attach this printed sheet to simplify your work.

**Problem Ex6.1-12. (Eigenpairs of a  $2 \times 2$ )**

Let  $A = \begin{pmatrix} 9 & -10 \\ 2 & 0 \end{pmatrix}$ . Find the eigenpairs of  $A$ . Then report eigenpair packages  $P$  and  $D$  such that  $AP = PD$ .

**Problem Ex6.1-20. (Eigenpairs of a  $3 \times 3$ )**

Let  $A = \begin{pmatrix} 5 & -6 & 3 \\ 6 & -7 & 3 \\ 6 & -6 & 2 \end{pmatrix}$ . Find the eigenpairs of  $A$ . Then report eigenpair packages  $P$  and  $D$  such that  $AP = PD$ .

**Problem Ex6.1-32. (Complex eigenpairs of a  $2 \times 2$ )**

Let  $A = \begin{pmatrix} 0 & -6 \\ 24 & 0 \end{pmatrix}$ . Find the eigenpairs of  $A$ . Then report eigenpair packages  $P$  and  $D$  such that  $AP = PD$ .

**Problem Ex6.1-36. (Eigenvalues of band matrices)**

Find the eigenvalues of the matrix  $A$  below without the aid of computers.

$$A = \begin{pmatrix} 1 & 2 & 0 & 0 & 0 & 0 \\ 2 & 1 & 2 & 0 & 0 & 0 \\ 0 & 2 & 1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 1 & 2 & 0 \\ 0 & 0 & 0 & 2 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

**Problem Ex6.2-6. (Eigenpair packages of a  $3 \times 3$ )**

Let  $A = \begin{pmatrix} 2 & -2 & 1 \\ 2 & -2 & 1 \\ 2 & -2 & 1 \end{pmatrix}$ . Find the eigenpairs of  $A$ . Then report eigenpair packages  $P$  and  $D$  such that  $AP = PD$ .

Check the answer by hand, expanding both products  $AP$  and  $PD$ , finally showing equality.

**Problem Ex6.2-18. (Fourier's model for a  $3 \times 3$ )**

Assume Fourier's model for a certain matrix  $A$ :

$$A \left( c_1 \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + c_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right) = 3c_1 \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + c_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

Find  $A$  explicitly from  $AP = PD$ . Check your answer by finding the eigenpairs of  $A$ .

**Problem Ex6.2-28. (Eigenpairs and diagonalization of a  $4 \times 4$ )**

Determine the eigenpairs of  $A$  below. If diagonalizable, then report eigenpair packages  $P$  and  $D$  such that  $AP = PD$ .

$$A = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 1 & 2 & 0 \\ 0 & 2 & 1 & 2 \\ 0 & 0 & 0 & 13 \end{pmatrix}$$

**End of extra credit problems chapter 6.**