Name ____________________________ Class Time _____

Math 2250 Extra Credit Problems
Chapter 3
February 2007

Due date: See the internet due date for 4.1, which is the due date for these problems. Records are locked on that date and only corrected, never appended.

Submitted work. Please submit one stapled package per problem. Kindly label problems **Extra Credit**. Label each problem with its corresponding problem number. You may attach this printed sheet to simplify your work.

Problem ExL2.1. (maple lab 2)

You may submit this problem only for score increases on maple lab 2.

Consider the linear differential equation $u' + ku = ka(t)$, $u(0) = u_0$, where $a(t) = 1 + \sin(\pi(t-3)/12)$. Solve the equation for $u(t)$ and check your answer in maple. Use maple assist for integration.

Problem ExL2.2. (maple lab 2)

You may submit this problem only for score increases on maple lab 2.

Consider the linear differential equation $u' + ku = ka(t)$, $u(0) = u_0$, where $a(t) = 1 + \sin(\pi(t-3)/12)$. Find the steady-state periodic solution of this equation and check your answer in maple.

Problem Ex3.1-16. (Elimination)

Solve the system below using frame sequences and report one of the three possibilities: no solution, unique solution, infinitely many solutions.

$$\begin{array}{rcrcrcl} x & + & 5y & + & 6z & = & 3, \\ 5x & + & 2y & - & 10z & = & 1, \\ 8x & + & 17y & + & 8z & = & 5. \end{array}$$

Problem Ex3.1-26. (systems of equations)

Give an example of a 3×3 system of equations which illustrates three planes, two of which intersect in a line, and that line lies entirely in the third plane.

Problem Ex3.2-14. (Echelon systems)

Solve the system below using frame sequences and report one of the three possibilities: no solution, unique solution, infinitely many solutions. Use variable list order x, y, z, w.

$$\begin{array}{rcrcrcrcl} 3x & - & 6y & + & z & + & 13w & = & 15, \\ 3x & - & 6y & + & 3z & + & 21w & = & 21, \\ 2x & - & 4y & + & 5z & + & 26w & = & 23. \end{array}$$

Problem Ex3.2-24. (Three possibilities with symbols)

Solve the system below for all values of a, b using frame sequences and report one of the three possibilities: no solution, unique solution, infinitely many solutions. If the system has a solution, then report the general solution.

$$\begin{array}{rcrcl} x & + & ay & = & b, \\ ax & + & (a-b)y & = & a. \end{array}$$

Problem Ex3.3-10. (RREF)

Show the frame sequence steps to $\mathbf{rref}(A)$ and attach a maple answer check (or do the whole problem in maple).

$$A = \begin{pmatrix} 1 & -4 & -2 \\ 3 & -12 & 1 \\ 2 & -8 & 5 \end{pmatrix}$$

Problem Ex3.3-20. (RREF)

Show the frame sequence steps to $\mathbf{rref}(A)$ and attach a maple answer check (or do the whole problem in maple).

$$A = \begin{pmatrix} 1 & -4 & -2 & 4 & 0 \\ 3 & -12 & 1 & 5 & 0 \\ 2 & -8 & 5 & 5 & 1 \end{pmatrix}$$

Problem Ex3.4-20. (Vector general solution)

Find the general solution in vector form $\mathbf{x}$, expressed as a linear combination of column vectors using symbols t_1, t_2, t_3 ... (as many symbols as needed for the free variables).

$$\begin{array}{rcrcrcrcrcl} x_1 & - & x_2 & & & + & 7x_4 & + & 3x_5 & = & 0, \\ & & & & x_3 & - & x_4 & - & 2x_5 & = & 0, \\ & & & & & & & & 0 & = & 0, \\ & & & & & & & & 0 & = & 0, \\ & & & & & & & & 0 & = & 0. \end{array}$$

Problem Ex3.4-40. (Superposition)

(a) Add the two systems below to prove that sums of solutions are again solutions. You will show that $\mathbf{x} = \begin{pmatrix} x_1 + x_2 \\ y_1 + y_2 \end{pmatrix}$ is a solution, given that $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ and $\begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$ are solutions of the homogeneous equation.

$$\left\{ \begin{array}{rcrcl} ax_1 & + & by_1 & = & 0, \\ cx_1 & + & dy_1 & = & 0. \end{array} \right. \qquad \left\{ \begin{array}{rcrcl} ax_2 & + & by_2 & = & 0, \\ cx_2 & + & dy_2 & = & 0. \end{array} \right.$$

(b) Add the two systems below to prove the superposition principle. You will show that $\mathbf{x} = \begin{pmatrix} x_1 + x_3 \\ y_1 + y_3 \end{pmatrix}$ is a solution, given that $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ solves the homogeneous problem and $\begin{pmatrix} x_3 \\ y_3 \end{pmatrix}$ solves the non-homogeneous problem.

$$\left\{ \begin{array}{rcrcl} ax_1 & + & by_1 & = & 0, \\ cx_1 & + & dy_1 & = & 0. \end{array} \right. \qquad \left\{ \begin{array}{rcrcl} ax_3 & + & by_3 & = & e, \\ cx_3 & + & dy_3 & = & f. \end{array} \right.$$

Problem Ex3.5-16. (Inverse by frame sequence)

Calculate the frame sequence from $C = ((: A), I)$ to $\mathbf{rref}(C)$ and report A^{-1}. Perform a hand answer check for the inverse matrix. No maple please, all with pencil and paper.

$$A = \begin{pmatrix} 1 & -2 & 2 \\ 3 & 0 & 1 \\ 1 & -1 & 2 \end{pmatrix}$$

Problem Ex3.5-44a. (Inverses and frame sequences)

(a) Suppose A is 8×8 and 60 entries are ones. Explain why A^{-1} does not exist.

(b) Suppose that A is invertible and 3×3. A frame sequence is started with A and gives final frame (not the **rref**)

$$\begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 3 \end{pmatrix}$$

The steps used to arrive at the final frame are (1) `combo(1,2,-3)`, (2) `swap(2,3)`, (3) `combo(1,2,-1)`, (4) `combo(2,3,1)`, (5) `mult(2,-1)`. Find the matrix A.

Problem Ex3.6-6. (Determinants and the four rules)

Calculate $\det(A)$ using only the four rules *triang, swap, combo, mult*. Check the answer in maple.

$$A = \begin{pmatrix} 1 & -4 & -2 & 4 & 0 \\ 3 & -12 & 1 & 5 & 0 \\ 2 & -8 & 5 & 5 & 1 \\ 0 & -8 & 5 & 5 & 1 \\ 0 & 0 & 5 & 5 & 1 \end{pmatrix}$$

Problem Ex3.6-20. (Determinants, hybrid rules)

Calculate $\det(A)$ using the four rules *triang, swap, combo, mult* plus the cofactor rule. Check the answer in maple.

$$A = \begin{pmatrix} 1 & -4 & -2 & 4 & 0 \\ 3 & -12 & 1 & 5 & 0 \\ 0 & -12 & 0 & 5 & 0 \\ 0 & -12 & 1 & 0 & 0 \\ 2 & -8 & 5 & 5 & 1 \end{pmatrix}$$

Problem Ex3.6-32. (Cramer's Rule)

Calculate x, y and z using Cramer's rule. Check the answer in maple.

$$\begin{pmatrix} 1 & -2 & 2 \\ 3 & 0 & 1 \\ 1 & -1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Problem Ex3.6-40. (Adjugate formula)

Find the inverse of the matrix A using the formula $A^{-1} = \dfrac{\text{adjugate}}{\text{determinant}}$.

$$A = \begin{pmatrix} 1 & -4 & -2 & 4 \\ 3 & -1 & 1 & 5 \\ 0 & -1 & 0 & 1 \\ 2 & 0 & -1 & 0 \end{pmatrix}$$

Problem Ex3.6-40. (Adjugate formula)

Find the entry in row 4 and column 2 of the adjugate matrix for A, using only determinants.

$$A = \begin{pmatrix} 1 & -4 & -2 & 4 \\ 3 & -1 & -1 & 3 \\ 0 & -1 & 0 & 1 \\ 2 & 0 & -1 & 0 \end{pmatrix}$$

Problem Ex3.6-60. **(Induction)**

Assume that $B_1 = 1$ and $B_2 = 2$. Assume $B_{k+2} = 2B_k + B_{k+1}$ for each integer $k = 1, 2, 3, \ldots$.

Let $\mathcal{Q}_n$ denote the statement that $B_k = 2^{k-1}$ for $1 \le k \le n$. Prove by mathematical induction that all statements $\mathcal{Q}_n$ are true.

Problem note: You must prove that $\mathcal{Q}_1$ and $\mathcal{Q}_2$ are true, individually. Mathematical induction then applies to the sequence of statements $\mathcal{Q}_3$, $\mathcal{Q}_4$, $\ldots$, in short, to statements $\mathcal{P}_j = \mathcal{Q}_{j+2}$, $j = 1, 2, 3, \ldots$.

End of extra credit problems chapter 3.