

**Math 2250 Extra Credit Problems**  
**Chapter 1**  
**January 2007**

**Due date:** See the internet due date for 2.1, which is the due date for these problems. Records are locked on that date and only corrected, never appended.

**Submitted work.** Please submit one stapled package per problem. Kindly label problems **Extra Credit**. Label each problem with its corresponding problem number. You may attach this printed sheet to simplify your work.

**Problem Ex1.2-4. (Quadrature)**

Solve  $y' = x^{-2} + x^{-1}$ ,  $y(1) = 2$ .

**Problem Ex1.2-10. (Quadrature)**

Solve  $y' = xe^{-2x} + x^2$ ,  $y(0) = 2$ .

**Problem Ex1.3-8. (Picard's theorem)**

Find a box with center  $x = 0$ ,  $y = 0$  to which Picard's theorem applies, verifying also continuity of  $f$  and  $f_y$  in the box, for the equation

$$y' = x\sqrt{x+y+1}, \quad y(0) = 0.$$

**Problem Ex1.3-14. (Peano's theorem)**

Does Peano's theorem apply to establish existence of at least one solution, for the problem below? Please carefully check the hypothesis of the theorem.

$$y' = 3(y-1)^{1/3}, \quad y(0) = 1.$$

**Problem Ex1.4-6. (Separable DE)**

Solve for equilibrium and non-equilibrium solutions (find the general solution).

$$y' = 2x \sec y.$$

**Problem Ex1.4-17. (Separability test)**

Use the test to verify that the equation  $y' = e^x + e^y$  is not separable.

**Problem Ex1.4-18. (Separability test)**

Find a factorization  $f(x, y) = F(x)G(y)$  for the problem below and then determine all non-equilibrium solutions.

$$y' = x^2(y^2 + y) + y^2 + x^2y + 2y + x^2 + 1.$$

**Problem Ex1.4-49. (Newton cooling)**

A roast is put into an oven whose temperature is  $400^\circ$  F. The meat thermometer was initially at  $40^\circ$  F and after 30 minutes it rose to  $90^\circ$  F. The roast is done when the thermometer reaches  $340^\circ$  F. How long does it take to cook the roast?

**Problem Ex1.5-4. (Linear DE)**

Solve  $y' - 2xy = e^{x^2}$ .

**Problem Ex1.5-16. (Linear DE)**

Solve  $y' = (1 - y) \cos x$ ,  $y(\pi) = 2$ .

**Problem Ex1.5-24. (Linear DE)**

Solve  $(x^2 + 4)y' + 3xy = x$ ,  $y(0) = 1$ .

**Problem Ex1.5-38. (Brine tank)**

Solve the brine tank problem

$$\begin{aligned}x'(t) &= -4x(t), \\y'(t) &= 4x(t) - 5y(t).\end{aligned}$$

**End of extra credit problems chapter 1.**