KEY

Differential Equations and Linear Algebra

2250-2 10:10am 3 May 2007

Instructions. The time allowed is 120 minutes. The examination consists of six problems, one for each of chapters 3, 4, 5, 6, 7, 10, each problem with multiple parts. A chapter represents 20 minutes on the final exam.

The final exam counts as two midterm exams. For example, if exam scores earned were 90, 91, 92 and the final exam score is 89, then the exam average for the course is

$$\frac{90 + 91 + 92 + 89 + 89}{5}.$$

Each problem represents several textbook problems numbered (a), (b), (c), $\cdots$. Choose the problems to be graded by check-mark $\boxed{\text{X}}$; the credits should add to 100. Each chapter (Ch3, Ch4, Ch5, Ch6, Ch7, Ch10) adds at most 100 towards the maximum final exam score of 600. The final exam score is reported as a percentage 0 to 100, which is the sum of the scores earned on six chapters divided by 600 to make a fraction, then converted to a percentage.

Calculators, books, notes and computers are not allowed.

Details count. Solutions requiring details are considered incomplete. Less than full credit is earned, in this case, for an answer only.

Answer checks are not expected or required. First drafts are expected, not complete presentations.

Please submit **exactly six** separately stapled packages of problems, one package per chapter.

Please discard this page or keep it for your records.

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Ch3. (Linear Systems and Matrices)

☐ [25%] Ch3(a): Let B be the invertible matrix given below, where $\boxed{?}$ means the value of the entry does not affect the answer to this problem. The second matrix C is the adjugate (or adjoint) of B. Find the value of $\det(2B^{-1}(CB)^2)$.

$$B = \begin{pmatrix} ? & ? & ? & 0 \\ 0 & -1 & 2 & 0 \\ 1 & 1 & 0 & 0 \\ ? & ? & ? & -3 \end{pmatrix}, \quad C = \begin{pmatrix} -6 & 3 & 9 & 0 \\ 6 & -3 & -6 & 0 \\ 3 & 0 & -3 & 0 \\ 2 & -1 & -3 & 1 \end{pmatrix}$$

☐ [25%] Ch3(b): State the three possibilities for a linear system $A\mathbf{x} = \mathbf{b}$ [5%]. Determine which values of k correspond to these three possibilities, for the system $A\mathbf{x} = \mathbf{b}$ given in the display below [20%].

$$A = \begin{pmatrix} 2 & 1 & -k \\ -4 & -1 & 2k-2 \\ -2 & -1 & k^2 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 2 \\ -3 \\ k-2 \end{pmatrix}$$

☐ [25%] Ch3(c): Assume matrix A is $n \times n$. State three different linear algebra theorems, each with the conclusion that $A\mathbf{x} = \mathbf{b}$ has infinitely many solution. No proofs, please!

☐ [25%] Ch3(d): Let A and B be two 3×3 upper triangular matrices with nonzero diagonal entries. Prove that $AB\mathbf{x} = \begin{pmatrix} 1 \\ -7 \\ 1000 \end{pmatrix}$ has a unique solution $\mathbf{x}$.

If you solved (a), (b), (c) and (d), then go on to Ch4. Otherwise, try (e) and (f). Maximum credit is 100%. Parts (e), (f) replace one of (a), (b), (c) or (d).

☐ [15%] Ch3(e): Find the value of x_2 by Cramer's Rule in the system $C\mathbf{x} = \mathbf{b}$, given C and $\mathbf{b}$ below. Evaluate determinants by any hybrid method (triangular, swap, combo, multiply, cofactor). The use of 2×2 Sarrus' rule is allowed. The 3×3 Sarrus' rule is **disallowed**.

$$C = \begin{pmatrix} -1 & 0 & -1 & 0 \\ 1 & 3 & -2 & 1 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 2 & 1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 0 \\ -1 \\ -1 \\ 1 \end{pmatrix}$$

☐ [10%] Ch3(f): Give an example of a matrix M such that $M\mathbf{x} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ has infinitely many solutions $\mathbf{x}$.

Staple this page to the top of all Ch3 work. Submit one package per chapter.

ch3 (a) $BC = \det(B) I$, $C = adj(B)$; $\det(B) = entry(2,2) = row(B,2)col(C,2)$
$= 3$; $\det(2\,B^{-1}(CB)^{-2}) = \det(2\,B^{-1}(3\,I)^{-2}) = \det(\frac{2}{9} I\, B^{-1})$
$= (\frac{2}{9})^4 \frac{1}{\det B} = \frac{2^4}{3^9}$

ch3 (b) 1. unique sol, 2. No sol, 3. ∞ - many sols

$$\left(\begin{array}{ccc|c} 2 & 1 & -k & 2 \\ -4 & -1 & 2k-2 & -3 \\ -2 & -1 & k^2 & k-2 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 2 & 1 & -k & 2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & k^2-k & k \end{array}\right)$$

1. unique sol for $k^2 - k \neq 0$ ($k \neq 0$ or 1)
2. No sol $k = 1$ (signal eq $0 = 1$)
3. ∞ - many sols $k = 0$ (one free var)

ch3 (c) Let $B = aug(A, \vec{b})$.

Thm 1. $A\vec{x} = \vec{b}$ consistent and $rank(B) < n$ $\Rightarrow$ ∞ - many sols.

Thm 2. $A\vec{x} = \vec{b}$ consistent and $\det(A) = 0$ $\Rightarrow$ ∞ - many sols.

Thm 3. $A\vec{x} = \vec{b}$ consistent and $rref(A) \neq I$ $\Rightarrow$ ∞ - many sols.

ch4 (d) Because $\det(E) =$ product of the diagonal elements for E triangular [triang rule], then $\det AB = \det A \det B \neq 0$. Then $\vec{x} = (AB)^{-1}\begin{pmatrix} -1 \\ 7 \\ 1000 \end{pmatrix}$ is the unique sol.

ch4 (e) $\det(C) = -12$, $\Delta_2 = 13$, $x_2 = -\frac{13}{12}$

ch4 (f) $\begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ has gen. sol. $\begin{cases} x_1 = 1 \\ x_2 = t_1 \end{cases}$ $-\infty < t_1 < \infty$

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Ch4. (Vector Spaces)

☐ [25%] Ch4(a): State (1) a rank test and (2) a determinant test to detect the independence or dependence of fixed vectors $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ [5%]. Apply one of the tests to the vectors below [10%]. Report which values of x make the set **independent** or **dependent** [10%].

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \\ x \\ 0 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} -1 \\ 0 \\ 3 \\ 1 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} -1 \\ 2 \\ 7x \\ 2 \end{pmatrix}.$$

☐ [25%] Ch4(b): Let $\mathbf{v}_1$, $\mathbf{v}_2$ be a basis for a subspace S of vector space $V = \mathcal{R}^n$. Let $\mathbf{w}_1$, $\mathbf{w}_2$ be any other proposed basis of S. Define E to be the augmented matrix of $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{w}_1$, $\mathbf{w}_2$. Does $\mathbf{rank}(E) = 2$ imply the bases are equivalent [10%]? Explain [15%].

☐ [50%] Ch4(c): Define the 4×4 matrix A by the display below. Find a basis of fixed vectors in $\mathcal{R}^4$ for (1) the column space of A [25%] and (2) the row space A [25%]. The two displayed bases **must** consist of columns of A and rows of A, respectively.

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 6 & 6 & 1 \\ 0 & -3 & -3 & 0 \\ 0 & -1 & -1 & 0 \end{pmatrix}$$

If you did (a), (b) and (c), then 100% has been marked – go on to Ch5. Otherwise, unmark one of (a), (b) or (c), then complete (d). Only three problems will be graded.

☐ [25%] Ch4(d): Define S to be the set of all vectors $\mathbf{x}$ in $\mathcal{R}^4$ such that $x_1 + 2x_4 = x_3$ and $x_2 = x_3 - x_4$. Prove or disprove that S is a subspace of $\mathcal{R}^4$.

Staple this page to the top of all Ch4 work. Submit one package per chapter.

Ch 4 (a) (1) v_1, v_2, v_3 independent $\Leftrightarrow$ rank(aug(v_1, v_2, v_3)) = 3

(2) $A = \text{aug}(v_1, v_2, v_3)$ 3×3. Then v_1, v_2, v_3 independent $\Leftrightarrow$ det(A) $\neq 0$.

Apply (1) to $U = \text{aug}(v_1, v_2, v_3)$. After combo + swap, $\begin{pmatrix} 1 & -1 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 6x-6 \\ 0 & 0 & 0 \end{pmatrix}$

The rank = 3 $\Leftrightarrow$ $x \neq 1$. Then $\{v_1, v_2, v_3\}$ indep $\Leftrightarrow$ $x \neq 1$.

Ch 4 (b) If the given vectors are a basis for S, then rank(aug(v_1, v_2)) = rank(aug(w_1, w_2)) = 2. By a standard Test, rank(F) = 2 implies equivalent bases.

Ch 4 (c) pivots of A = 2, 4; Pivots of A^T = 2, 3

Colspace(A) = span{col(A,2), col(A,4)}

rowspace(A) = span{row(A,2), row(A,3)}

Ch 4 (d) Define $A = \begin{pmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$. The equations are equivalent to $A\vec{x} = \vec{0}$. Apply the kernel Theorem, $S = \{\vec{x} : A\vec{x} = \vec{0}\}$ is a subspace.

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Ch5. (Linear Equations of Higher Order)

☐ [10%] Ch5(a): Find the general solution of the differential equation

$$15y'' + 44y' + 21y = 0.$$

☐ [20%] Ch5(b): Find the homogeneous differential equation general solution, given characteristic equation

$$(r^2 + 5r)^3(r^4 + 5r^2)(r^2 + 10r + 21)^2 = 0.$$

☐ [10%] Ch5(c): Given a damped spring-mass system $mx''(t) + cx'(t) + kx(t) = 0$ with $m = 3$, $c = 16$ and $k = 21$, classify the answer as over-damped, critically damped or under-damped. Please, **do not solve** the differential equation!

☐ [40%] Ch5(d): Assume a 12th order constant-coefficient differential equation has characteristic equation $r^2(r^2 + 4)^2(r^2 + 16)^3 = 0$. Suppose the right side of the differential equation is

$$f(x) = x(x^2 + 2\sin 2x) + x\cos 4x$$

Determine the **corrected** trial solution for y_p according to the method of undetermined coefficients. **Do not evaluate** the undetermined coefficients!

☐ [20%] Ch5(e): Find the steady-state periodic solution for the equation

$$9x'' + 6x' + 5x = 4930\cos(5t).$$

Staple this page to the top of all Ch5 work. Submit one package per chapter.

Ch 5 (a) $y(x) = c_1 e^{-3x/5} + c_2 e^{-7x/3}$

Ch 5 (b) $r^3(r+5)^3(r^2)(r^2+5)(r+7)^2(r+3)^2 = 0$

atoms $= 1, x, x^2, x^3, x^4, \quad e^{-5x}, x e^{-5x}, x^2 e^{-5x},$
$\cos\sqrt{5}x, \sin\sqrt{5}x, \ e^{-7x}, x e^{-7x}, \ e^{-3x}, x e^{-3x}$
$= 14$ atoms

$y_h(x) =$ linear combination of the 14 atoms, using $c_1 \longrightarrow c_{14}$

Ch 5 (c) $3r^2 + 16r + 21 = 0$
$r = -7/3, \ r = -3$ over-damped

Ch 5 (d) roots of char eq $= 0, 0, 2i, 2i, -2i, -2i, 4i, 4i, 4i, -4i, -4i, -4i$

trial sol $= y = x^2(d_1 + d_2 x + d_3 x^2 + d_4 x^3)$
$+ x^2(d_5 \cos 2x + d_6 x \cos 2x + d_7 \sin 2x + d_8 x \sin 2x)$
$+ x^3(d_9 \cos 4x + d_{10} x \cos 4x + d_{11} \sin 4x + d_{12} x \sin 4x)$

Ch 5 (e) $x_{ss}(t) = -22\cos(5t) + 3\sin(5t)$
Start with $x = d_1 \cos 5t + d_2 \sin 5t$. Then apply undetermined coefficients to get

$$\begin{cases} -220 d_1 + 30 d_2 = 4930 \\ -30 d_1 - 220 d_2 = 0 \end{cases} \quad \text{or} \quad \begin{cases} -22 d_1 + 3 d_2 = 493 \\ -3 d_1 - 22 d_2 = 0 \end{cases}$$

Because $22^2 + 9 = 493$, then $d_1 = -22, d_2 = 3$.

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Ch6. (Eigenvalues and Eigenvectors)

☐ [25%] Ch6(a): Find the eigenvalues of the matrix

$$A = \begin{pmatrix} 0 & 12 & -3 & 0 \\ 12 & 0 & -6 & 3 \\ 0 & 0 & -5 & 2 \\ 0 & 0 & 2 & -5 \end{pmatrix}.$$

To save time, **do not** find eigenvectors!

☐ [25%] Ch6(b): Let $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$. Prove or disprove: Fourier's model holds for A.

☐ [25%] Ch6(c): Describe how to find two different 2×2 matrices A and B having eigenvalues $\lambda_1 = 1$, $\lambda_2 = 2$, such that each matrix has one of its eigenpairs equal to

$$\left(1, \begin{pmatrix} 1 \\ 2 \end{pmatrix}\right).$$

☐ [25%] Ch6(d): The matrix A below has eigenvalue package

$$D = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

State whether A is diagonalizable, and if it is, then display the matrix package P of eigenvectors.

$$A = \begin{pmatrix} 4 & 0 & 0 \\ -1 & 4 & 1 \\ 1 & 0 & 3 \end{pmatrix}$$

If you did (a), (b), (c) and (d), then 100% has been marked – go on to Ch7. Otherwise, unmark one of (a), (b), (c) or (d), then complete (e). Only four problems will be graded.

☐ [25%] Ch6(e): Assume two 3×3 matrices A and B have exactly the same characteristic equations. Let A have eigenvalues 3, 4, 4. Find the eigenvalues of $(1/3)B - 2I$, where I is the identity matrix.

Staple this page to the top of all Ch6 work. Submit one package per chapter.

ch 6 (a) By cofactor expansion, $\det(A-\lambda I) = (\lambda+3)(\lambda+7)(\lambda+12)(\lambda-12)$

eigenvalues $= -3, -7, -12, 12$

ch 6 (b) By inspection $\lambda_1 = \lambda_2 = \lambda_3 = 0$. Then $A - \lambda I = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix} \cong \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

All eigenvectors are $t_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$. So there is no Fourier model.

ch 6 (c) Let $P_1 = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$, $P_2 = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$, $D = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$.

Define $A = P_1 D P_1^{-1}$, $B = P_2 D P_2^{-1}$. Then $A \neq B$ because they have different eigenpairs.

ch 6 (d) Eigenpairs $= \left(4, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}\right), \left(4, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right), \left(3, \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}\right)$

$P = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ or $\begin{pmatrix} 0 & 0 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$. must be in matching pairs.

ch 6 (e) Let $C = \frac{1}{3} B - 2I$. Then

$$\begin{aligned}
\det(C - \lambda I) &= \det\left(\tfrac{1}{3} B - 2I - \lambda I\right) \\
&= \det\left(\tfrac{1}{3} I (B - 6I - 3\lambda I)\right) \\
&= \det\left(\tfrac{1}{3} I\right) \det(B - \mu I) \qquad \mu = 6 + 3\lambda
\end{aligned}$$

Then $\det(C - \lambda I) = 0 \Leftrightarrow \lambda = \frac{\mu - 6}{3}$ where $\mu =$ an eigenvalue of $B =$ an eigenvalue of $A = 3, 4, 4$

$\frac{3-6}{3}, \frac{4-6}{3}, \frac{4-6}{3} =$ eigenvalues of $C = -1, \frac{-2}{3}, \frac{-2}{3}$

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Ch7. (Linear Systems of Differential Equations)

☐ [25%] Ch7(a): Apply the eigenanalysis method to solve the system $\mathbf{x}' = A\mathbf{x}$, given

$$A = \begin{pmatrix} 6 & -2 & -4 \\ 0 & 3 & 0 \\ 8 & -7 & -6 \end{pmatrix}.$$

☐ [25%] Ch7(b): Give an example of a 2×2 real non-triangular matrix A for which Fourier's model is valid. Then display the general solution $\mathbf{x}(t)$ of $\mathbf{x}' = A\mathbf{x}$.

☐ [25%] Ch7(c): Solve the triangular system below.

$$\begin{aligned} x' &= x + 5y, \\ y' &= y. \end{aligned}$$

☐ [25%] Ch7(d): Let A be a general 2×2 real matrix. State completely two different theorems which could apply to solve the differential equation $\mathbf{x}' = A\mathbf{x}$.

If you solved (a), (b), (c) and (d), then you have marked 100%. If so, then go on to Ch10, otherwise, continue here. Only 4 parts will be graded.

☐ [25%] Ch7(e): Consider a 3×3 system $\mathbf{x}' = A\mathbf{x}$. Assume A has eigenvalues $\lambda_1 = -2$, $\lambda_2 = -3 + i$, $\lambda_3 = -3 - i$. Prove that $\lim_{t\to\infty} \|\mathbf{x}(t)\| e^t = 0$ for all solutions $\mathbf{x}(t)$ of the differential equation.

Staple this page to the top of all Ch7 work. Submit one package per chapter.

ch 7 (a) Eigenpairs of $A = \left(-2, \begin{pmatrix}1\\0\\2\end{pmatrix}\right), \left(2, \begin{pmatrix}1\\0\\1\end{pmatrix}\right), \left(3, \begin{pmatrix}2\\1\\1\end{pmatrix}\right)$

$$\vec{x}(t) = c_1 e^{-2t}\begin{pmatrix}1\\0\\2\end{pmatrix} + c_2 e^{2t}\begin{pmatrix}1\\0\\1\end{pmatrix} + c_3 e^{3t}\begin{pmatrix}2\\1\\1\end{pmatrix}$$

ch 7 (b) $A = PDP^{-1} = \begin{pmatrix}0 & 1\\ -2 & 3\end{pmatrix}$ where $P = \begin{pmatrix}1 & 1\\ 1 & 2\end{pmatrix}$, $D = \begin{pmatrix}1 & 0\\ 0 & 2\end{pmatrix}$.

Then $\left(1, \begin{pmatrix}1\\1\end{pmatrix}\right), \left(2, \begin{pmatrix}1\\2\end{pmatrix}\right)$ are eigenpairs of A and Fouriers model holds. Finally, $\vec{x}(t) = c_1 e^{t}\begin{pmatrix}1\\1\end{pmatrix} + c_2 e^{2t}\begin{pmatrix}1\\2\end{pmatrix}$.

ch 7 (c) $y' = y$ has sol $y = c_2 e^t$. Then $x' = x + 5c_2 e^t$.
By the linear integrating factor method,

$$\begin{cases} x(t) = 5c_2 t e^t + c_1 e^t \\ y(t) = c_2 e^t \end{cases}$$

ch 7 (e) By the theory,

$$\vec{x}(t) = c_1 e^{-2t}\vec{v}_1 + c_2 e^{-3t} e^{it}\vec{v}_2 + c_3 e^{-3t} e^{-it}\vec{v}_3$$

Then each term of $e^t \vec{x}(t)$ contains a negative exponential factor, which implies

$$\lim_{t\to\infty} e^t \vec{x}(t) = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

Then

$$\lim_{t\to\infty} e^t \|\vec{x}(t)\| = 0$$

ch 7 (d)

Thm 1 If A is triangular, then solve $x' = Ax$ by first solving the growth-decay equation, then substitute the answer into the other equation and solve by the linear integrating factor method

Thm 2 Eigenanalysis method

Thm 3 Find the roots of $\det(A - \lambda I) = 0$. Then both $x(t)$, $y(t)$ are linear combinations of the atoms for these roots. If A is not triangular, then this result solves $x' = Ax$, A 2×2 real.

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Ch10. (Laplace Transform Methods)

It is assumed that you have memorized the basic 4-item Laplace integral table and know the 6 basic rules for Laplace integrals. No other tables or theory are required to solve the problems below. If you don't know a table entry, then leave the expression unevaluated for partial credit.

☐ [25%] Ch10(a): Apply Laplace's method to the system to find a 2×2 system for $\mathcal{L}(x)$, $\mathcal{L}(y)$. To save time, **do not solve** the system and don't find $x(t)$ or $y(t)$!

$$\begin{aligned} x'' &= x + y, \\ y'' &= -x + 2y, \\ x(0) &= 1, \quad x'(0) = 0, \\ y(0) &= 0, \quad y'(0) = 0. \end{aligned}$$

☐ [25%] Ch10(b): Solve for $f(t)$, given

$$\mathcal{L}(f(t)) = \left(\left(\frac{d}{ds}\right)^2 \left(\mathcal{L}(e^t \sin t)\right) \right)\Bigg|_{s \to (s+2)} + \frac{s+s^2}{(s+2)^4} + \mathcal{L}\left(1 + te^{-2t}\right)$$

☐ [25%] Ch10(c): Find $f(t)$ by partial fraction methods, given

$$\mathcal{L}(f(t)) = \frac{2+3s}{s^3+2s^2} + \frac{s^2-3}{(s+1)^2(s^2+1)}.$$

☐ [25%] Ch10(d): Apply Laplace's method to find a formula for $\mathcal{L}(x(t))$. To save time, **do not** solve for $x(t)$! Document steps by reference to tables and rules.

$$x''' + 5x'' = t^2 + e^t(1 + e^t \sin 2t), \quad x(0) = 1, \quad x'(0) = x''(0) = 0.$$

Staple this page to the top of all Ch10 work. Submit one package per chapter.

Ch10 (a) $\begin{cases} (s^2-1)\mathcal{L}(x) + (-1)\mathcal{L}(y) = s \\ (1)\,\mathcal{L}(x) + (s^2-2)\mathcal{L}(y) = 0 \end{cases}$

Ch10 (b) $\mathcal{L}(f_1) = \left(\left(\frac{d}{ds}\right)^2 \mathcal{L}(e^t \sin t)\right)\Big|_{s\to s+2}$

$= \mathcal{L}((-t)(-t)e^t \sin t)\ |_{s\to s+2}$

$= \mathcal{L}(t^2 e^{-2t} e^t \sin t)$

$f_1 = t^2 e^{-t} \sin t$

$\mathcal{L}(f_2) = \dfrac{s+s^2}{(s+2)^4} = \dfrac{s-2+(s-2)^2}{s^4}\Big|_{s\to s+2}$

$= \dfrac{s^2-3s+2}{s^4}\Big|_{s\to s+2}$

$= \mathcal{L}(t - \frac{3}{2}t^2 + 2\frac{t^3}{6})\ |_{s\to s+2}$

$f_2 = (t - \frac{3}{2}t^2 + \frac{1}{3}t^3)e^{-2t}$

$f_3 = 1 + t e^{-2t}$ [Third term was finished]

$\mathcal{L}(f) = \mathcal{L}(f_1) + \mathcal{L}(f_2) + \mathcal{L}(f_3) \Rightarrow f = f_1 + f_2 + f_3$ by Lerch's thm.

Ch10 (c) $\dfrac{2+3s}{s^3+2s^2} = \dfrac{1}{s} + \dfrac{1}{s^2} - \dfrac{1}{s+2} = \mathcal{L}(1 + t - e^{-2t})$

$\dfrac{s^2-3}{(s+1)^2(s^2+1)} = \dfrac{-2}{s+1} + \dfrac{-1}{(s+1)^2} + \dfrac{2s}{s^2+1} = \mathcal{L}(-2e^{-t} - te^{-t} + 2\cos t)$

$f = 1 + t - e^{-2t} - 2e^{-t} - te^{-t} + 2\cos t$

Ch10 (d) $-s^2 + s^3 \mathcal{L}(x) + 5(-1 + s^2 \mathcal{L}(x)) = \mathcal{L}(t^2 + e^t(1 + e^t \sin 2t))$

$= \dfrac{2}{s^3} + \dfrac{1}{s-1} + \dfrac{2}{(s-2)^2+4}$

$\mathcal{L}(x) = \dfrac{s^2+5s + \frac{2}{s^3} + \frac{1}{s-1} + \frac{2}{(s-2)^2+4}}{s^3+5s^2}$