Math 2250
Earthquake project
April 2004

Name ______________________________________ Class Time __________

Project 3. Solve problems 3.1 to 3.6. The problem headers:

_______ PROBLEM 3.1. BUILDING MODEL FOR AN EARTHQUAKE.
_______ PROBLEM 3.2. TABLE OF NATURAL FREQUENCIES AND PERIODS.
_______ PROBLEM 3.3. UNDETERMINED COEFFICIENTS STEADY-STATE PERIODIC SOLUTION.
_______ PROBLEM 3.4. PRACTICAL RESONANCE.
_______ PROBLEM 3.5. EARTHQUAKE DAMAGE.
_______ PROBLEM 3.6. SIX FLOORS.

3.1. BUILDING MODEL FOR AN EARTHQUAKE.
Refer to the textbook of Edwards-Penney, section 7.4, page 437.
Consider a building with 7 floors.

Let the mass in slugs of each story be m=1000.0 and let the spring constant be k=10000.0 (lbs/foot). Define the 7 by 7 mass matrix M and Hooke's matrix K for this system and convert Mx''=Kx into the system x''=Ax where A is defined by textbook equation (1) , page 437.

PROBLEM 3.1
Find the eigenvalues of the matrix A to six digits, using the Maple command "eigenvals(A)." Answer check: All seven eigenvalues should be negative.

```
 # Sample Maple code for a model with 4 floors.
 # Use maple help to learn about evalf and eigenvals.
  with(linalg):
  A := matrix([ [-20,10,0,0], [10,-20,10,0], [0,10,-20,10],
 [0,0,10,-10]]);
  evalf(eigenvals(A),6);
>
> # Problem 3.1
> with(linalg):
>
```

3.2. TABLE OF NATURAL FREQUENCIES AND PERIODS.
Refer to figure 7.4.17, page 437.

PROBLEM 3.2.
Find the natural angular frequencies omega=sqrt(-lambda) for the seven story building and also the corresponding periods 2PI/omega, accurate to six digits. Display the answers in a simple handwritten table or a computer-generated table as in the example below. The answers appear in Figure 7.4.17, page 437, although in a slightly different order than what would be computed in MAPLE.

```
 # Sample code for a  4x3  table.
 # Use maple help to learn about nops and printf.
  ev:=[-10,-1.206147582,-35.32088886,-23.47296354]:
  n:=nops(ev):
  Omega:=lambda -> sqrt(-lambda):
  format:="%10.6f  %10.6f  %10.6f\n":
  printf("%s    %s        %s\n","Eigenvalue", "Freq","Period");
  seq(printf(format,ev[i],Omega(ev[i]),2*evalf(Pi)/Omega(ev[i])),i=1..n
 );

> # Problem 3.2

> with(linalg):
>
```

3.3. UNDETERMINED COEFFICIENTS STEADY-STATE PERIODIC SOLUTION.
Consider the forced equation x''=Ax+cos(wt)b where b is a constant vector. The earthquake's ground vibration is accounted for by the extra term cos(wt)b, which has period T=2Pi/w. The solution x(t) is the 7-vector of excursions from equilibrium of the corresponding 7 floors. Sought here is not the general solution, which certainly contains transient terms, but rather the steady-state periodic solution, which is known from the theory to have the form x(t)=cos(wt)c for some vector c that depends only on A and b. See the textbook, page 433.

PROBLEM 3.3.
Define b:=0.25*w*w*vector([1,1,1,1,1,1,1]): in Maple and find the vector c in the undetermined coefficients solution x(t)=cos(wt)c. Vector c depends on w. As outlined in the textbook, vector c can be found by solving the linear algebra problem -w^2 c = Ac + b; see page 433. Don't print c, as it does not fit on one page; instead, print c[1] as an illustration. You should get -0.09304656278 when c[1] is evaluated at w=1.

```
 # Sample code for defining b and A, then solving for c in the 4-floor
 # case. See maple help to learn about vector and linsolve.
   w:='w': u:=w*w: b:=0.25*u*vector([1,1,1,1]):
   Au:=matrix([ [-20+u,10,0,0], [10,-20+u,10,0], [0,10,-20+u,10],
              [0,0,10,-10+u]]);
   c:=linsolve(Au,-b):
   'c[1]'=evalf(c[1],2);
   subs(w=1,evalf(c[1],2));

> # PROBLEM 3.3
> with(linalg):
> # subs(w=1,evalf(c[1],2));
>
```

3.4 PRACTICAL RESONANCE.
Consider the forced equation x'=Ax+cos(wt)b of 3.3 above with b:=0.25*w*w*vector([1,1,1,1,1,1,1]). Practical resonance can occur if a component of x(t) has large amplitude compared to the vector norm of b.

For example, an earthquake might cause a small 3-inch excursion on level ground, but the building's floors might have 50-inch excursions, enough to destroy the building.

PROBLEM 3.4.
Let Max(c) denote the maximum modulus of the components of vector c. Plot g(T)=Max(c(w)) with w=(2*Pi)/T for periods T=0 to T=4, ordinates Max=0 to Max=10, the vector c(w) being the answer produced in 3.3 above. Compare your figure to the textbook Figure 7.4.18, page 438. Your figure is expected to show 6 spikes.

```
# Sample maple code to define the function Max(c), 4-floor building.
# Use maple help to learn about norm, vector, subs and linsolve.
  with(linalg):
  w:='w': Max:= c -> norm(c,infinity); u:=w*w:
  b:=0.25*w*w*vector([1,1,1,1]):
  Au:=matrix([ [-20+u,10,0,0], [10,-20+u,10,0], [0,10,-20+u,10],
[0,0,10,-10+u]]);
  C:=ww -> subs(w=ww,linsolve(Au,-b)):
  plot(Max(C(2*Pi/r)),r=0..4,0..10,numpoints=400);
```

```
> # PROBLEM 3.4. WARNING: Save your file often!!!
> with(linalg):
> # plot(Max(C(2*Pi/r)),r=0..4,0..10,numpoints=400);
>
```

3.5. EARTHQUAKE DAMAGE.
The maximum amplitude plot of 3.4 can be used to detect the likelihood of earthquake damage for a given ground vibration of period T. A ground vibration (1/4)cos(wt), T=2*Pi/w, will be assumed, as in 3.4.

PROBLEM 3.5.
(a) Replot the amplitudes in 3.4 for graph window x=0.95 to 3.5 and y=5 to 10. There will be six spikes.
(b) Create one zoom-in plot near T=3, choosing a T-interval that shows the full spike.
(c) Determine from the zoom-in plot (near T=3) one approximate T-interval such that some floor in the building will undergo excursions from equilibrium in excess of 5 feet.

```
# Example: Zoom-in on a spike for amplitudes 5 feet to 10 feet,
# periods 1.96 to 2.01.
with(linalg): w:='w': Max:= c -> norm(c,infinity); u:=w*w:
Au:=matrix([ [-20+u,10,0,0], [10,-20+u,10,0], [0,10,-20+u,10],
[0,0,10,-10+u]]);
b:=0.25*w*w*vector([1,1,1,1]):
C:=ww -> subs(w=ww,linsolve(Au,-b)):
plot(Max(C(2*Pi/r)),r=1.96..2.01,5..10,numpoints=400);
printf("Period T from 1.96 to 2.01");
```

```
> # PROBLEM 3.5. WARNING: Save your file often!!
```

```
> #(a) Plot six spikes on one graph
> #(b) Plot one zoom-in graph near T=3.
> #(c)  Report one approximate T-interval  near T=3.
>
```

3.6. SIX FLOORS.
Consider a building with six floors each weighing 50 tons. Assume each floor corresponds to a restoring Hooke's force with constant k=10 tons/foot. Assume that ground vibrations from the earthquake are modeled by (1/4)cos(wt) with period T=2*Pi/w (same as the 7-floor model above).

PROBLEM 3.6.
Model the 6-floor problem in Maple. Plot the maximum amplitudes in graph window x=1 to 4 and y=4 to 10. Determine from the graphic one period range near T=3.5 which causes the amplitude plot to spike above 4 feet.

Sanity checks: Recall that a ton equals 2000 pounds, and that a pound of force equals mass (in slugs) times the acceleration of gravity, 32 ft/sec/sec. From this you can work out how to convert tons to slugs. Use (5) and (6) page 425 to find the matrices M and K (on paper) and then write down A as the inverse of M times K. Check your reasoning on the original model: your logic should reproduce the text before equation (1) page 437. There are five spikes. To see them, follow the examples above, especially, use the plot option numpoints=400 or larger.

```
> # PROBLEM 3.6. WARNING: Save your file often!!
> # Define k, m and the 6x6 matrix A.
> # Amplitude plot for T=1..4,C=4..10
> # Plot one zoom-in graphic near T=3.5.
> # Print the T-range near T=3.5 for the zoom-in plot above.
>
```