Math 2250

Earthquake project

November 2003

Name _____ Class Time _____
Project 3. Solve problems 3.1 to 3.6. The problem headers:

_____ PROBLEM 3.1. BUILDING MODEL FOR AN EARTHQUAKE.
_____ PROBLEM 3.2. TABLE OF NATURAL FREQUENCIES AND PERIODS.
_____ PROBLEM 3.3. UNDETERMINED COEFFICIENTS STEADY-STATE PERIODIC SOLUTION.
_____ PROBLEM 3.4. PRACTICAL RESONANCE.
_____ PROBLEM 3.5. EARTHQUAKE DAMAGE.
_____ PROBLEM 3.6. SIX FLOORS.
> with(linalg):

3.1. BUILDING MODEL FOR AN EARTHQUAKE.

Refer to the textbook of Edwards-Penney, section 7.4, page 437. Consider a building with 7 floors.

Let the mass in slugs of each story be $m = 1000.0$ and let the spring constant be $k = 10000$ lbs/foot. Define the 7 by 7 mass matrix M and Hooke's matrix K for this system and convert $Mx'' = Kx$ into the system $x'' = Ax$ where A is defined by textbook equation (1) , page 437.

PROBLEM 3.1
Find the eigenvalues of the matrix A to six digits, using the Maple command $\boxed{\texttt{eigenvals(A)}}$. Check that that all seven eigenvalues are negative by using maple's $\boxed{\texttt{eigenvals}}$ command.
Sample Maple code for a model with 4 floors.
Use maple help to learn about evalf and eigenvals.
with(linalg):
A:=matrix([[-20,10,0,0], [10,-20,10,0], [0,10,-20,10], [0,0,10,-10]]);
evalf(eigenvals(A),6);
> # Problem 3.1

3.2. TABLE OF NATURAL FREQUENCIES AND PERIODS.
Refer to figure 7.4.17, page 437.

PROBLEM 3.2.
Find the natural angular frequencies $\omega = \sqrt{-\lambda}$ for the seven story building and also the corresponding periods $2\pi/\omega$, accurate to six digits. Display the answers in a table . The answers appear in Figure 7.4.17, page 437.
Sample code for a 4x3 table.
Use maple help to learn about nops and printf.
ev:=[-10,-1.206147582,-35.32088886,-23.47296354]: n:=nops(ev):
Omega:=lambda -> sqrt(-lambda):
format:="%10.6f %10.6f %10.6f\n":
printf("%s   %s   %s\n","Eigenvalue", "Freq","Period");
seq(printf(format,ev[i],Omega(ev[i]),2*evalf(Pi)/Omega(ev[i])),i=1..n);

```
>    # Problem 3.2
```

3.3. UNDETERMINED COEFFICIENTS STEADY-STATE PERIODIC SOLUTION.
Consider the forced equation $x'' = Ax + \cos(wt)b$ where b is a constant vector. The earthquake's ground vibration is accounted for by the extra term $\cos(wt)b$, which has period $T = 2\pi/w$. The solution x(t) is the 7-vector of excursions from equilibrium of the corresponding 7 floors. Sought here is not the general solution, which certainly contains transient terms, but rather the steady-state periodic solution, which is known from the theory to have the form $x(t) = \cos(wt)c$ for some vector c that depends only on A and b.

PROBLEM 3.3.
Define `b:=0.25*w*w*vector([1,1,1,1,1,1,1]):` in Maple and find the vector c in the undetermined coefficients solution $x(t) = \cos(wt)c$. Vector c depends on w. As outlined in the textbook, vector c can be found by solving the linear algebra problem $-w^2c = Ac + b$; see page 433. Don't print c, as it is too complex; instead, print `c[1]` as an illustration.
Sample code for defining b and A, then solving for c in the 4-floor case.
See maple help to learn about vector and linsolve.
with(linalg):
w:='w': u:=w*w: b:=0.25*u*vector([1,1,1,1]):
Au:=matrix([[-20+u,10,0,0], [10,-20+u,10,0], [0,10,-20+u,10], [0,0,10,-10+u]]);
c:=linsolve(Au,-b):
'c[1]'=evalf(c[1],2);
```
>    # PROBLEM 3.3
```

3.4 PRACTICAL RESONANCE.
Consider the forced equation $x'' = Ax + \cos(wt)b$ of 3.3 above. Practical resonance can occur if a component of $x(t)$ has large amplitude compared to the vector norm of b. For example, an earthquake might cause a small 3-inch excursion on level ground, but the building's floors might have 50-inch excursions, enough to destroy the building.

PROBLEM 3.4.
Let Max(c) denote the maximum modulus of the components of vector c. Plot $g(T) = Max(c(w))$ with $w = 2\pi/T$ for periods $T = 0$ to $T = 6$, ordinates $Max = 0$ to $Max = 10$, the vector $c(w)$ being the answer produced in 3.3 above. Compare your figure to the textbook Figure 7.4.18, page 438. Adjust parameter `numpoints` in the plot command as needed.
Sample maple code to define the function Max(c), 4-floor building.
Use maple help to learn about norm, vector, subs and linsolve.
with(linalg):
w:='w': Max:= c -> norm(c,infinity); u:=w*w:
b:=0.25*w*w*vector([1,1,1,1]):
Au:=matrix([[-20+u,10,0,0], [10,-20+u,10,0], [0,10,-20+u,10], [0,0,10,-10+u]]);
C:=ww -> subs(w=ww,linsolve(Au,-b)):
plot(Max(C(2*Pi/r)),r=0..6,0..10,numpoints=150);
```
>    # PROBLEM 3.4.   WARNING: Save your file often!!!
```

3.5. EARTHQUAKE DAMAGE.
The maximum amplitude plot of 3.4 can be used to detect the likelihood of earthquake damage for a given ground vibration of period T. A ground vibration $(1/4)\cos(wt)$, $T = 2\pi/w$, will be assumed, as in 3.4.

PROBLEM 3.5.

(a) Replot the amplitudes in 3.4 for periods 1.14 to 4 and amplitudes 5 to 10. There will be four spikes.

(b) Create four zoom-in plots, one for each spike, choosing a T-interval that shows the full spike.

(c) Determine from the four zoom-in plots approximate intervals for the period T such that some floor in the building will undergo excursions from equilibrium in excess of 5 feet. Adjust parameter $\boxed{\texttt{numpoints}}$ in the plot command as needed.

\# Example: Zoom-in on a spike for amplitudes 5 feet to 10 feet, periods 1.97 to 2.01.

with(linalg): w:='w': Max:= c -> norm(c,infinity); u:=w*w:

Au:=matrix([[-20+u,10,0,0], [10,-20+u,10,0], [0,10,-20+u,10], [0,0,10,-10+u]]);

b:=0.25*w*w*vector([1,1,1,1]):

C:=ww -> subs(w=ww,linsolve(Au,-b)):

plot(Max(C(2*Pi/r)),r=1.97..2,01,5..10,numpoints=150);

```
>   # PROBLEM 3.5.   WARNING: Save your file often!!
>   #(a) Plot four spikes on separate graphs
>   #(b) Plot four zoom-in graphs.
>   #(c) Print period ranges.
```


3.6. FIVE FLOORS.

Consider a building with five floors each weighing 40 tons. Assume each floor corresponds to a restoring Hooke's force with constant k=5 tons/foot. Assume that ground vibrations from the earthquake are modeled by (1/4)cos(wt) with period T=2*Pi/w (same as the 7-floor model above).

PROBLEM 3.6.

Model the 5-floor problem in Maple. Plot the maximum amplitudes against the period 1 to 15 and amplitude 4 to 10. Determine from the graphic the period ranges which cause the amplitude plot to spike above 4 feet.

Sanity checks: Recall that a ton equals 2000 pounds, and that a pound of force equals mass (in slugs) times the acceleration of gravity, 32 ft/sec/sec. From this you can work out how to convert tons to slugs. You can check your reasoning on the original model, see text before equation (1) page 437. There are five spikes, but you may have to zoom in to see them.

```
>   # PROBLEM 3.6.   WARING: Save your file often!!
>   # Define k, m and the 5x5 matrix A.
>   # Amplitude plot for T=0..15,C=4..10
>   # Plot five zoom-in graphs
>   # From the graphics, five T-ranges give amplitude # spikes above 4 feet.  These are determined
    by left # mouse-clicks on the graph, so they are approximate values only.
```