Syllabus for the Differential Equations Preliminary Examination

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Instructions: The examination consists of two parts. **Part A** consists of exercises concerning *Ordinary differential Equations* and **Part B** consists of exercises concerning *Partial Differential Equations*.

To obtain full credit, please complete three exercises from part A and three exercises from part B, a total of six (6) exercises. All exercises are equally weighted and partial credit applies to each. A passing score is 60% of the total possible score.

Sound and detailed solutions are expected, but bear in mind that too many details are time-consuming. Judgement of what is essential will be an important factor in determining the final score.

When asked to *outline a proof*, it means to write a sequence of statements and lemmas with very brief details. Otherwise, full detail is expected, however brief it has to be. This does not mean that major steps are skipped: it could mean a lemma is stated and not proved, whereas in regular coursework extra pages of details would appear.

Part A
Ordinary Differential Equations
Do three (3) exercises from Part A for full credit.

Exercise A-1. Existence-uniqueness theory for initial value problems, fixed point theory, Lebesgue integration theory. Reference: Schmitt-Thompson. Select with a check–mark $\boxed{\surd}$ and solve either A-1.I or A-1.II:

☐ A-1.I.

☐ A-1.II.

Exercise A-2. Matrix linear systems and stability. Periodic solutions of linear nonhomogeneous systems. Reference: Schmitt-Thompson. Select with a check–mark $\boxed{\surd}$ and solve either A-2.I or A-2.II:

☐ A-2.I. Parts (a), (b).

☐ A-2.II.

Exercise A-3. Flow for autonomous systems. Limit sets, Poincaré-Bendixon theory, Floquet theory. Rest points, linearization, phase diagram, geometric classifications of rest points. Periodic orbits, limit cycles. Reference: Schmitt-Thompson. Select with a check–mark $\boxed{\surd}$ and solve either A-3.I or A-3.II:

☐ A-3.I. Parts (a), (b), (c).

☐ A-3.II.

Exercise A-4. Extension of solutions and dependence on parameters. Topological degree. Reference: Schmitt-Thompson. Select with a check–mark $\boxed{\surd}$ and solve either A-4.I or A-4.II:

☐ A-4.I: Parts (a), (b), (c).

☐ A-4.II: Parts (a), (b), (c).

Part B
Partial Differential Equations
Do three (3) problems from Part B for full credit.

Exercise B-1. Sturm–Liouville theory, including separation and comparison theory, eigenfunction expansion theory, and the related Hilbert space theory such as orthogonality and completeness. Explicit calculation of eigenvalues and eigenfunctions. Parts (a), (b), (c). References: Schmitt-Thompson, Showalter.

Exercise B-2. Sobolev space theory for H^m, Hilbert space theory, distributional derivatives, Riesz theory. References: Schmitt-Thompson, Showalter. Select with a check–mark ☑ and solve either B-2.I or B-2.II:

☐ B-2.I: Parts (a), (b), (c).

☐ B-2.II: Parts (a), (b).

Exercise B-3. Distributional differential equations, abstract boundary value problem, sesquilinear forms, Riesz theory, Dirichlet and Neumann problems, Poincaré inequality, Hilbert space and distributional solutions. References: Schmitt-Thompson and Showalter. Select with a check–mark ☑ and solve either B-3.I or B-3.II:

☐ B-3.I: Parts (a), (b), (c).

☐ B-3.II: Parts (a), (b).

Exercise B-4. Sobolev embedding inequalities, distributional solutions, smooth solutions, regularity theory, abstract boundary value problems, classical solutions to the Dirichlet and Neumann problems. Reference: Showalter. Select with a check–mark ☑ and solve either B-4.I or B-4.II:

☐ B-4.I: Parts (a), (b).

☐ B-4.II: Parts (a), (b).