Name. _____

Differential Equations 5420
Midterm Exam 2, Spring 2003
Due Date: March 14, 2003

Instructions. The three problems below are take-home, due on the date above. Answer checks are expected. If `maple` assist is used, then please attach the `maple` output.

1. **(Three methods)** Solve the system of differential equations $x' = Ax$ by each of the three methods: (a) Laplace, (b) matrix exponential, (c) eigenanalysis.

$$A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 3 \\ 0 & 1 & -1 \end{pmatrix}$$

2. **(Forced system)** Solve $x' = Ax + F(t)$. Maple assist expected for integration.

$$A = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 2 & 3 & 2 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad F(t) = \begin{pmatrix} 1 \\ 0 \\ -1 \\ t+1 \end{pmatrix}.$$

3. **(Stability)** Determine the stability properties of the trivial solution of $x' = Ax$, given

$$A = \begin{pmatrix} -3 & -2 & 1 & 0 \\ 1 & -1 & 0 & 1 \\ 0 & 0 & -3 & -2 \\ 0 & 0 & 1 & -1 \end{pmatrix}.$$