Exact Solution 2.4,2.5,2.6-#12

The exact symbolic solution for the initial value problem should be derived just once for 2.4-#12, and then only referenced in 2.5-#12, 2.6-#12. The derivation of the symbolic answer is collected in class as a regular assignment, not as a part of this lab. Please do not duplicate the derivation in the report for lab 3. The answer:

$$y = \frac{x-4}{x-2}.$$

A table of exact values is required in order to make comparison tables. Make this table for each problem separately, as the values used vary from one comparison to another.

2.4 Notes

Numerical Solution 2.4-#12

This work has to be done before you can write the report. Please write a report that references an appendix to be attached as a worksheet print; see below for the content of the appendix. Include here handwritten material that describes the Euler algorithm as applied to problem #12, then reference the worksheet for results.

The maple code referenced in the internet document *Numerical DE Manuscript* will be used. There is a text file of the actual code segments in the internet document document *Numerical DE maple coding hints*. Both are located at the course web site. The basic maple code:

```
# Warning: These snips of code made for y'=1-x-y, y(0)=3.
#          Code computes approx values for y(0.01) to y(1.0).
# 'Dots' is the list of dots for connect-the-dots graphics.
# =======================================
# Euler. Group 1, initialize.
 f:=(x,y)->1-x-y:
 x0:=0:y0:=3:h:=0.01:Dots:=[x0,y0]:
# Group 2, repeat 100 times. Euler's method
 for j from 1 to 100 do
 Y:=y0+h*f(x0,y0);
 x0:=x0+h:y0:=Y:Dots:=Dots,[x0,y0];
 end do:
# Group 3, show Dots, then plot.
 Dots[1],Dots[21],Dots[41],Dots[61],Dots[81],Dots[101];
 plot([Dots]);
```

To start, get the sample Euler code to produce correct answers to the example supplied in the text file source. Once correct, modify the code for #12. The step size $h = 0.01$ produces a dot table of 101 rows, whereas the step size $h = 0.005$ makes a dot table with 201 rows. Please do not attach these tables to the maple worksheet appendix! They are too large and also uninteresting. An abbreviated table will be made.

Comparison Table 2.4-#12

The comparison will be 6 rows in 2.4-#12, which means only some of the dot table data is used in the report. The table should list x, $y1$, $y2$, y, E where $y1$ is the $h = 0.01$ approximate value, $y2$ is the $h = 0.005$ approximate value, y is the exact value and E is the relative percentage error of the more accurate approximation

$$E = 100|y - y2|/|y|.$$

The code segments in the internet document document *Numerical DE maple coding hints* describe how to wrap the main loop of the algorithm in a loop, and how to extract just the relevant data from the large dots table. The coding hints also discuss how to write maple code for the error value above.

Graphics 2.4-#12

There should be three separate graphics, one for $h = 0.01$, one for $h = 0.005$ and one for the exact solution. All are produced in maple. Reference the maple worksheet appendix.

Appendix: Hand Solution Steps 2.4-#12

Include a derivation of the numerical values for line two of the dot table for each case $h = 0.01$ and $h = 0.005$. Show all steps by hand. This is the only cross-check on the numerics.

Appendix: Maple Worksheet 2.4-#12

Attach a print of the maple worksheet that contains all computer code and data used in 2.4-#12. Reference this appendix while creating the report.

2.5 Notes

Numerical Solution 2.5-#12

This work has to be done before you can write the report. Please write a report that references an appendix to be attached as a worksheet print; see below for the content of the appendix. Include here handwritten material that describes the Heun (modified Euler) algorithm as applied to problem #12, then reference the worksheet for results.

Sample Heun code:

```
# Warning: These snips of code made for y'=1-x-y, y(0)=3.
#         Code computes approx values for y(0.01) to y(1.0).
# 'Dots' is the list of dots for connect-the-dots graphics.
# =====================================
# Heun [=Modified Euler]. Group 1, initialize.
 f:=(x,y)->1-x-y:
 x0:=0:y0:=3:h:=0.01:Dots:=[x0,y0]:
# Group 2, repeat 100 times. Heun's method
 for j from 1 to 100 do
 Y1:=y0+h*f(x0,y0);
 Y:=y0+h*(f(x0,y0)+f(x0+h,Y1))/2:
 x0:=x0+h:y0:=Y:Dots:=Dots,[x0,y0];
 end do:
# Group 3, show Dots, then plot.
 Dots[1],Dots[21],Dots[41],Dots[61],Dots[81],Dots[101];
 plot([Dots]);
```

To start, get the sample Heun code to produce correct answers to the example supplied in the text file source. Once correct, modify the code for #12. The step size $h = 0.01$ produces a dot table of 101 rows whereas step size $h = 0.005$ produces 201 rows. Neither of these dot tables should appear in your report - leave them un-printed in the maple appendix. Instead, a data compression scheme will be introduced to print just the relevant portions of the dot tables (see group 3 in the example).

The code segments in the internet document document *Numerical DE maple coding hints* describe how to wrap the main loop of the algorithm in a loop, and how to extract just the relevant data from the large dots table. The coding hints also discuss how to write maple code for the error value.

Comparison Table 2.5-#12

The comparison will be 6 rows in 2.5-#12. The table should list x, $y1$, $y2$, y, E where x is one of the values 1, 1.2, 1.4, 1.6, 1.8, 2.0, and $y1$ is the $h = 0.01$ approximate value, $y2$ is the $h = 0.005$ approximate value, y is the exact value and E is the percentage relative error measured by

$$E = 100|y - y2|/|y|.$$

The code segments in the internet document document *Numerical DE maple coding hints* show how to code the above error value. Also described therein is how to print just the relevant data. DO NOT print the entire dot table – it is too large.

Graphics 2.5-#12

There should be three graphics, one for $h = 0.01$, one for $h = 0.005$ and one for the exact solution. All are produced in maple. Reference the maple worksheet appendix. You may used compressed data for the dot tables in order to speed the graphics production.

Appendix: Hand Solution Steps 2.5-#12

Include a derivation of the numerical values for line two of the dots table for $h = 0.01$. Show all steps by hand. This is the only cross-check on the numerics. Do not show any details for $h = 0.005$.

Appendix: Maple Worksheet 2.5-#12

Attach a print of the maple worksheet that contains all computer code and data used in 2.5-#12. Reference this appendix during the production of the report.

2.6 Notes

Numerical Solution 2.6-#12

This work has to be done before you can write the report. Please write a report that references an appendix to be attached as a worksheet print; see below for the content of the appendix. Include here handwritten material that describes the RK4 algorithm as applied to problem #12, then reference the worksheet for results. The code segments in the internet document document *Numerical DE maple coding hints* show how to code the RK4 algorithm. Do not include computer code here – display the mathematical formulas that will be used in the maple code, as applied to the present problem.

Sample RK4 code:

```
# Warning: These snips of code made for y'=1-x-y, y(0)=3.
#          Code computes approx values for y(0.01) to y(1.0).
# 'Dots' is the list of dots for connect-the-dots graphics.
# =======================================
# RK4. Group 1, initialize.
 f:=(x,y)->1-x-y:
 x0:=0:y0:=3:h:=0.2:Dots:=[x0,y0]:
# Group 2, repeat 5 times. RK4 method
 for j from 1 to 5 do
 k1:=h*f(x0,y0):
 k2:=h*f(x0+h/2,y0+k1/2):
 k3:=h*f(x0+h/2,y0+k2/2):
 k4:=h*f(x0+h,y0+k3):
 Y:=y0+(k1+2*k2+2*k3+k4)/6:
 x0:=x0+h:y0:=Y:Dots:=Dots,[x0,y0];
 end do:
# Group 3, show Dots, then plot.
```

```
   Dots[1],Dots[2],Dots[3],Dots[4],Dots[5],Dots[6];
   plot([Dots]);
```

To start, get the sample RK4 code to produce correct answers to the example supplied in the text file source. Once correct, modify the code for #12. The step size $h = 0.2$ produces a dot table of 6 rows and step size $h = 0.1$ makes a dot table of 11 rows.

Comparison Table 2.6-#12

The comparison will be 6 rows in 2.6-#12. The table should list x, $y1$, $y2$, y, E where x is 1.0, 1.2, 1.4, 1.6, 1.8, 2.0, and $y1$ is the $h = 0.2$ approximate value, $y2$ is the $h = 0.1$ approximate value, y is the exact value and E is the percentage relative error of the more accurate approximation

$$E = 100|y - y2|/|y|.$$

The code segments in the internet document document *Numerical DE maple coding hints* show how to code the above error value. Extraction of the correct data items from the two dot tables is also illustrated there.

Graphics 2.6-#12

There should be three graphics, one for $h = 0.2$, one for $h = 0.1$ and one for the exact solution. All are produced in maple. Reference the maple worksheet appendix.

Appendix: Hand Solution Steps 2.6-#12

Skip this step for 2.6-#12, because the machine is likely more reliable than a hand calculation. Instead of a hand check, check the algorithm on several problems which have known solutions.

Appendix: Maple Worksheet 2.6-#12

Attach a print of the maple worksheet that contains all computer code and data used in 2.6-#12. Reference this appendix during the production of the report.