

Name \_\_\_\_\_ Class Time \_\_\_\_\_

Project 8. Solve problems L8.1 to L8.5. The problem headers:

----- PROBLEM L8.1. EARTHQUAKE MODEL FOR A BUILDING.  
----- PROBLEM L8.2. TABLE OF NATURAL FREQUENCIES AND PERIODS.  
----- PROBLEM L8.3. UNDETERMINED COEFFICIENTS STEADY-STATE SOL  
----- PROBLEM L8.4. PRACTICAL RESONANCE.  
----- PROBLEM L8.5. EARTHQUAKE DAMAGE.

FIVE FLOOR Model.

Refer to the textbook of Edwards-Penney, section 7.4, page 437.  
Consider a building with five floors each weighing 50 tons. Each floor corresponds to a restoring Hooke's force with constant  $k=5$  tons/foot. Assume that ground vibrations from the earthquake are modeled by  $(1/4)\cos(\omega t)$  with period  $T=2\pi/\omega$ .

PROBLEM L8.1. BUILDING MODEL FOR AN EARTHQUAKE.

Model the 5-floor problem in Maple.

Define the 5 by 5 mass matrix  $M$  and Hooke's matrix  $K$  for this system and convert  $Mx''=Kx$  into the system  $x''=Ax$  where  $A$  is defined by textbook equation (1), page 437.

Sanity check: Mass  $m=3125$ , and the 5x5 matrix contains fraction  $16/5$ .

Then find the eigenvalues of the matrix  $A$  to six digits, using the Maple command "eigenvals(A)."

Sanity check: All six eigenvalues should be negative.

```
# Sample Maple code for a model with 4 floors.  
# Use maple help to learn about evalf and eigenvals.  
# A:=matrix([ [-20,10,0,0], [10,-20,10,0],  
[0,10,-20,10],[0,0,10,-10]]);  
# with(linalg): evalf(eigenvals(A));
```

```
# Problem L8.1  
# Define k, m and the 5x5 matrix A.  
# with(linalg): evalf(eigenvals(A));
```

PROBLEM L8.2. TABLE OF NATURAL FREQUENCIES AND PERIODS.

Refer to figure 7.4.17, page 437.

Find the natural angular frequencies  $\omega=\sqrt{-\lambda}$  for the five story building and also the corresponding periods  $2\pi/\omega$ , accurate to six digits. Display the answers in a table. Compare with answers in Figure 7.4.17, page 437, for the 7-story case.

```
# Sample code for a 4x3 table, 4-story building.  
# Use maple help to learn about nops and printf.
```

```

# ev:=[-10,-1.206147582,-35.32088886,-23.47296354]: n:=nops(ev):
# Omega:=lambda -> sqrt(-lambda):
# format:="%10.6f %10.6f %10.6f\n":
# seq(sprintf(format,ev[i],Omega(ev[i]),2*evalf(Pi)/Omega(ev[i])),
i=1..n);

# Problem L8.2
# ev:=[fill this in]: n:=nops(ev):
# Omega:=lambda -> sqrt(-lambda): format:="%10.6f %10.6f %10.6f\n":
#
seq(sprintf(format,ev[i],Omega(ev[i]),2*evalf(Pi)/Omega(ev[i])),i=1..n)
;

```

### PROBLEM L8.3. UNDETERMINED COEFFICIENTS

#### STEADY-STATE PERIODIC SOLUTION.

Consider the forced equation  $x' = Ax + \cos(\omega t)b$  where  $b$  is a constant vector. The earthquake's ground vibration is accounted for by the extra term  $\cos(\omega t)b$ , which has period  $T = 2\pi/\omega$ .

The solution  $x(t)$  is the 5-vector of excursions from equilibrium of the corresponding 5 floors.

Sought here is not the general solution, which certainly contains transient terms, but rather the steady-state periodic solution, which is known from the theory to have the form  $x(t) = \cos(\omega t)c$  for some vector  $c$  that depends only on  $A$  and  $b$ .

Define  $b := 0.25 * \omega * \omega * \text{vector}([1,1,1,1,1])$ : in Maple and find the vector  $c$  in the undetermined coefficients solution  $x(t) = \cos(\omega t)c$ . Vector  $c$  depends on  $\omega$ . As outlined in the textbook, vector  $c$  can be found by solving the linear algebra problem  $-\omega^2 c = Ac + b$ ; see page 433. Don't print  $c$ , as it is too complex; instead, print  $c[1]$  as an illustration.

```

#Sample code for defining b and A, then solving for c
#in the 4-floor case.
# See maple help to learn about vector and linsolve.
# w:='w': u:=w*w: b:=0.25*u*vector([1,1,1,1]):
# A:=matrix([ [-20,10,0,0], [10,-20,10,0],
[0,10,-20,10], [0,0,10,-10]]);
# Au:=evalm(A+u*diag(1,1,1,1));
# c:=linsolve(Au,-b):
# evalf(c[1],2);

```

### # PROBLEM L8.3

```

# Define w, u, b, A, Au, c
# evalf(c[1],2);

```

### PROBLEM L8.4. PRACTICAL RESONANCE.

Consider the forced equation  $x' = Ax + \cos(\omega t)b$  of L8.3 above with  $b := 0.25 * \omega * \omega * \text{vector}([1,1,1,1,1])$ .

Practical resonance can occur if a component of  $x(t)$  has large amplitude compared to the vector norm of  $b$ . For example, an earthquake might cause a small 3-inch excursion on level ground, but the building's floors might have 50-inch excursions, enough to destroy

the building.

Let  $\text{Max}(c)$  denote the maximum modulus of the components of vector  $c$ . Plot  $g(T)=\text{Max}(c(w))$  with  $w=(2\pi)/T$  for periods  $T=1$  to  $T=5$ , ordinates  $\text{Max}=0$  to  $\text{Max}=10$ , the vector  $c(w)$  being the answer produced in L8.3 above. Compare your figure to the textbook Figure 7.4.18, page 438.

```
# Sample maple code to define the function Max(c), 4-floor building.
# Use maple help to learn about norm, vector, subs and linsolve.
# with(linalg):
# w:='w': Max:= c -> norm(c,infinity); u:=w*w:
# b:=0.25*w*w*vector([1,1,1,1]):
# A:=matrix([ [-20,10,0,0], [10,-20,10,0], [0,10,-20,10],
[0,0,10,-10]]);
# Au:=evalm(A+u*diag(1,1,1,1));
# C:=ww -> subs(w=ww,linsolve(Au,-b)):
# plot(Max(C(2*Pi/r)),r=1..5,0..10,numpoints=150);

# PROBLEM L8.4. WARNING: Save your file often!!!
# w:='w': Max:= c -> norm(c,infinity): u:=w*w:
# Define b
# Define A
# Define Au
# Define C
# plot(Max(C(2*Pi/r)),r=1..5,0..10,numpoints=150);
```

#### PROBLEM L8.5. EARTHQUAKE DAMAGE.

The maximum amplitude plot of L8.4 can be used to detect the of earthquake damage for a given ground vibration of period  $T$ . A ground vibration  $(1/4)\cos(wt)$ ,  $T=2\pi/w$ , will be assumed, as in L8.4.

- Replot the amplitudes in L8.4 for periods 1.5 to 5.5 and amplitudes 5 to 10. There will be several spikes.
- Create several zoom-in plots, one for each spike, choosing a  $T$ -interval that shows the full spike.
- Determine from the several zoom-in plots approximate intervals for the period  $T$  such that some floor in the building will undergo excursions from equilibrium in excess of 5 feet.

```
# Example: Zoom-in on a spike for amplitudes 5 feet to 10 feet,
#periods 1.97 to 2.01. This example for the 4-floor problem.
#with(linalg): w:='w': Max:= c -> norm(c,infinity); u:=w*w:
#Au:=matrix([ [-20+u,10,0,0], [10,-20+u,10,0],
[0,10,-20+u,10],[0,0,10,-10+u]]);
#b:=0.25*w*w*vector([1,1,1,1]):
#C:=ww -> subs(w=ww,linsolve(Au,-b)):
#plot(Max(C(2*Pi/r)),r=1.97..2,01,5..10,numpoints=150);

# PROBLEM L8.5. WARNING: Save your file often!!
#(a) Re-plot the five spikes.
# plot(Max(C(2*Pi/r)),r=1.5..5.5,5..10,numpoints=150);
```

```
#(b) Plot five zoom-in graphs.
# one:=1.79..1.83:plot(Max(C(2*Pi/r)),r=one,5..10,numpoints=150);
# two:=???:plot(Max(C(2*Pi/r)),r=two,5..10,numpoints=150);
# three:=???:plot(Max(C(2*Pi/r)),r=three,5..10,numpoints=150);
# four:=???:plot(Max(C(2*Pi/r)),r=four,5..10,numpoints=150);
# five:=???:plot(Max(C(2*Pi/r)),r=five,5..10,numpoints=150);
#(c) Print period ranges.
# PeriodRanges:=[one,two,three,four,five];
```