Math 2250 Maple Lab 6, F2010. Mechanical Oscillations.

NAME _____ CLASSTIME ____

There are three (3) problems in this project. Please answer the
questions A, B, C , ... associated with each problem. The original
worksheet "2250mapleL6-F2010.mws" is a template for the solution; you
must fill in the code and all comments. Sample code can be copied with
the mouse. Use pencil freely to annotate the worksheet and to clarify
the code and figures.

The problem headers for the Spring 2010 version of Mechanical
Oscillations maple lab 6.

_____L6.1. UNDER-DAMPED FREE OSCILLATIONS.
_____L6.2. UNDAMPED FORCED OSCILLATIONS.
_____L6.3. PRACTICAL RESONANCE.

L6.1. PROBLEM (UNDER-DAMPED FREE OSCILLATIONS)

FREE OSCILLATIONS. Consider the problem of free linear oscillations

          m x'' + c x' + k x=0,
          x(0)=0, x'(0)=1.

Here, m, c and k are non-negative constants. The under-damped case is
studied here, c^2 < 4km, as on page 327 in E&P. Assume m=4 and c=3.

   A. Display a Hooke's constant k > 0 so that the solution x(t) is
      under-damped. Check that x(t)=0 for infinitely many t>0.
      Display the exact solution x(t) obtained by maple methods
      as in the example below.

   B. Plot the exact symbolic solution x(t) on a suitable t-interval.
      Check the graphic against Figure 5.4.9 page 328 of E&P.

   C. Estimate from the graph the decimal value of the pseudo-period.
      Display the graphical estimate and also the exact pseudo-period
      2Pi/w, where w is the natural frequency of the trigonometric term
      in the solution x(t) found above in item L6.1-A.

EXAMPLE(Wrong parameters! Change it!)
Use semicolons to see what you have done.
Define the differential equation
de:=3*diff(x(t),t,t)+1.5*diff(x(t),t)+4*x(t)=0:
Solve the characteristic equation.
solve(3*r^2+1.5*r+4=0,r);
Define the initial conditions
ic:=x(0)=0,D(x)(0)= 1:
Symbolically solve for x(t)
p:=dsolve({de,ic},x(t),method=laplace):
Capture the dsolve symbolic solution as a function X(t)
X:=unapply(rhs(p),t):
Plot the solution
plot(X(t),t=0..5);

Maple tip: Click with the mouse on the graphic to print the cursor
location (left upper corner of the maple window). The coordinates
printed are of the form (x,y). From this coordinate information, a
simple subtraction estimates the period.

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#L6.1-A Define  k, then solve.
# under-damped means mr^2+cr+k=0 has two conjugate complex roots.
#L6.1-B Plot.
#L6.1-C Pseudo-period calculations.
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L6.2. PROBLEM (UNDAMPED FORCED OSCILLATIONS)

FORCED LINEAR OSCILLATIONS. Consider the undamped (c=0) forced
problem

```
    mx'' +  k x = 5 cos(wt),
    x(0)=0,   x'(0)=0,
```

where m, k and w are non-negative constants. Assume m=5 and k=3.5,

   A. Choose the forcing angular frequency w to be 3 times larger than
       the natural angular frequency w0, w0^2=k/m. Solve for x(t) using
       dsolve(). Plot the solution x(t) on a suitable interval in order
       to show the global behavior of the solution x(t). See Figure
       5.6.2, page 350.

   B. The solution x(t) is the sum of two functions, one of period
      2Pi/w and the other of period 2Pi/w0. Display the exact period,
      as calculated from the solution formula for x(t) -- see page
      350 for details.

   C. Suggest a value for the forcing frequency w so that the
      oscillations exhibit resonance. Show resonant behavior on a
      graph. Check against Figure 5.6.4, page 352.


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#L6.2-A
#L6.2-B
#L6.2-C
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L6.3. PROBLEM (PRACTICAL RESONANCE)
  Consider the damped forced problem

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    mx'' + c x' + k x = 5 cos(w t),
    x(0)=0,   x'(0)=0.
```

Assume m=4, k=41.

   A. Consider the damping constants c=2, c=1 and c=1/2. Compute the
      amplitude function C(w) [page 357] for these three equations,
      then plot for w=0 to w=20 the three amplitude graphs on a single
      set of axes. Compare against Figure 5.6.9 page 357 of E&P
      (it has one curve, yours has 3 curves).

B. For each case c=2, c=1, c=1/2, print the values w*, C* where
C*=C(w*)=max {C(w) : 0 <= w <= 20}. The three data pairs should
show that C* becomes larger as c tends to zero. SAVE YOUR MAPLE
FILE FREQUENTLY

Maple Hint: Use Maple's mouse interface on the graphic of Part C.
Specifically, click on a possible maximum (horizontal tangent) in
the graph to display the values w*, C* on the screen. Look around
the screen to see where maple printed the x,y-coordinates! Copy the
values into your maple worksheet report.

```
#EXAMPLE(Beware! Wrong values!)
#F:=15: m:=1: k:=25: c:='c': w:='w':
#C:=(w,c)->F/sqrt((k-m*w*w)^2+(c*w)^2):
#plot({C(w,4),C(w,3),C(w,2)},w=0..15,color=black);

#L6.3-A Plot C(w), three graphics on one set of axes
#L6.3-B Table of six data values for w*, C*
```