Name ____________________ Class Time _____

Math 2250 Extra Credit Problems
Chapter 10
Fall 2010

Due date: Submit these problems one week after 10.4 is submitted. Records are locked on that date and only corrected, never appended. The scores on Ch10 extra credit can replace any missing score for Chapter 10.

Submitted work. Please submit one stapled package with this sheet on top. Kindly check-mark the problems submitted and label the problems **Extra Credit**. Label each problem with its corresponding problem number, e.g., Xc10.3-20.

Problem Xc10.3-20. (Inverse transform)

Solve for $f(t)$ in the relation $\mathcal{L}(f(t)) = \dfrac{1}{s^4 - 8s^2 + 16}$. Use partial fractions in the details.

Problem Xc10.3-24. (Inverse transform)

Solve for $f(t)$ in the relation $\mathcal{L}(f(t)) = \dfrac{s}{s^4 + 4a^4}$, showing the details that give the answer $f(t) = \dfrac{1}{2a^2} \sinh at \sin at$

Problem Xc10.4-12. (Inverse transform, convolution)

Solve for $f(t))$ in the relation $\mathcal{L}(f(t)) = \dfrac{1}{s(s^2 + 4s + 5)}$. Instead of the convolution theorem, use partial fractions for the details. If you can see how, then check the answer with the convolution theorem.

Problem Xc10.4-26. (Inverse transform techniques)

Use the s-differentiation theorem in the details of solving for $f(t)$ in the relation $\mathcal{L}(f(t)) = \arctan \dfrac{3}{s+2}$. You will need to apply the theorem $\lim_{s\to\infty} \mathcal{L}(f(t)) = 0$.

Problem Xc10.4-40. (Series methods for transforms)

Expand in a series, using Taylor series formulas, the function $f(t) = \dfrac{\cos 2\sqrt{t}}{\sqrt{\pi t}}$. Then find $\mathcal{L}(f(t))$ as a series by Laplace transform of each series term, separately. Finally, re-constitute the series in variable s into elementary functions, namely $e^{-1/s}$ divided by $\sqrt{s}$.

Problem Xc10.5-6. (Second shifting theorem, Heaviside step)

Find the function $f(t)$ in the relation $\mathcal{L}(f(t)) = \dfrac{se^{-s}}{s^2 + \pi^2}$.

Problem Xc10.5-14. (Transforms of piecewise functions)

Let $f(t) = \begin{cases} \cos \pi t & 0 \le t \le 2, \\ 0 & t > 2. \end{cases}$ Find $\mathcal{L}(f(t))$. Details should expand $f(t)$ as a linear combination of Heaviside step functions.

Problem Xc10.5-26. (Sawtooth wave)

Let $f(t + a) = f(t)$ and $f(t) = t$ on $0 \le t \le a$. Then f is a-periodic and has a Laplace transform obtained from the periodic function formula. Show the details in the derivation to obtain the answer $\mathcal{L}(f(t)) = \dfrac{1}{as^2} - \dfrac{e^{-as}}{s(1 - e^{-as})}$.

Problem Xc10.5-28. (Modified sawtooth wave)

Let $f(t + 2a) = f(t)$ and $f(t) = t$ on $0 \le t \le a$, $f(t) = 0$ on $a < t \le 2a$. Then f is $2a$-periodic and has a Laplace transform obtained from the periodic function formula. Derive a formula for $\mathcal{L}(f(t))$. The answer to this problem can be found in Edwards-Penney, section 10.5.

Problem Xc-EPbvp-7.6-8. **(Impulsive DE)**

Solve by Laplace methods $x'' + 2x' + x = \delta(t) - 2\delta(t-1)$, $x(0) = 1$, $x'(0) = 1$. Check the answer in `maple` using `dsolve({de,ic},x(t),method=laplace)`.

Problem Xc-EPbvp-7.6-18. **(Switching circuit)**

A passive LC-circuit has battery 6 volts and model equation $i'' + 100i = 6\delta(t) - 6\delta(t-1)$, $i(0) = 1$, $i'(0) = 1$. The switch is closed at time $t = 0$ and opened again at $t = 1$. Solve the equation by Laplace methods and report the number of full cycles observed before the steady-state $i = 0$ is reached (to two decimal places). Check the answer in `maple` using `dsolve({de,ic},i(t),method=laplace)`.

End of extra credit problems chapter 10.