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# Partial Differential Equations 3150

**Sample Midterm Exam 1**

**Exam Date: Tuesday, 27 October 2009**

**Instructions**: This exam is timed for 50 minutes. You will be given double time to complete the exam. No calculators, notes, tables or books. Problems use only chapters 1 and 2 of the textbook. No answer check is expected. Details count 3/4, answers count 1/4.

1. **(Vibration of a Finite String)**
   Some **normal modes** for the string equation $u_{tt} = c^2 u_{xx}$ are given by the equation

$$u(x,t) = \sin\left(\frac{n\pi x}{L}\right)\cos\left(\frac{n\pi ct}{L}\right).$$

   (a) [25%] Give an example of a finite linear combination of normal modes.

   (b) [25%] Write a mathematical argument, using the superposition principle, showing that the example given in (a) is a solution of $u_{tt} = c^2 u_{xx}$.

   (c) [50%] Solve the finite string vibration problem on $0 \le x \le 1$, $t > 0$,

$$\begin{aligned} u_{tt} &= c^2 u_{xx}, \\ u(0,t) &= 0, \\ u(1,t) &= 0, \\ u(x,0) &= 2\sin(\pi x) - 3\sin(5\pi x), \\ u_t(x,0) &= 0. \end{aligned}$$

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**2. (Periodic Functions)**

(a) [25%] Find the period of $f(x) = \sin 2x \cos 2x$.

(b) [25%] Give an example of a piecewise continuous function on $0 \le x \le 2$ that has a discontinuity at $x = 1$.

(c) [25%] Is $f(x) = \cos(2x + 3)$ an even periodic function?

(d) [25%] Is $f(x) = \sin(\pi x/5)$ an odd periodic function?

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**3. (Fourier Series)**
Let $f(x) = 1$ on the interval $0 < x < 2\pi$, $f(x) = -1$ on $-2\pi < x < 0$, $f(x) = 0$ for $x = 0, 2\pi, -2\pi$. Let $g(x)$ be the $4\pi$-periodic extension of $f$ to the whole real line.

(a) [25%] Is $g(x)$ even or odd?

(b) [25%] Display the formulas for the Fourier coefficients of $f$.

(c) [25%] Compute the Fourier coefficient for the term $\sin(5x)$.

(d) [25%] Are there any values of $x$ such that $g(x)$ does not equal the Fourier series of $f$?

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**4. (Cosine and Sine Series)**
Find the first three terms in the cosine series expansion of the cosine wave $g(x)$, formed as the even periodic extension of the base function $\cos x + 2\cos 4x$ on $0 < x < \pi$.

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**5. (Convergence of Fourier Series)**

(a) [25%] Display Dirichlet's kernel formula.

(b) [25%] State the Fourier Convergence Theorem for piecewise smooth functions.

(c) [25%] Fourier convergence may not be uniform, and the commonly referenced term to describe this problem is Gibb's phenomenon. Explain what it is, by example.

(d) [25%] State Parseval's identity for complex Fourier series.

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