Frame Sequences with Symbol k

Math 2250 Fall 2007

October 5, 2007

Example: Three Possibilities with Symbol k

Determine all values of the symbol k such that the system below has one of the **Three Possibilities** (1) *No solution*, (2) *Infinitely many solutions* or (3) *A unique solution*. Display all solutions found.

$$\begin{array}{rcrcl} x & + & ky & = & 2, \\ (2-k)x & + & y & = & 3. \end{array}$$

Example: Three Possibilities with Symbol k

Determine all values of the symbol k such that the system below has one of the **Three Possibilities** (1) *No solution*, (2) *Infinitely many solutions* or (3) *A unique solution*. Display all solutions found.

$$\begin{array}{rcrcl} x & + & ky & = & 2, \\ (2-k)x & + & y & = & 3. \end{array}$$

The Three Possibilities are detected by (1) A signal equation "$0 = 1$," (2) One or more free variables, (3) Zero free variables.

Example: Three Possibilities with Symbol k

Determine all values of the symbol k such that the system below has one of the **Three Possibilities** (1) *No solution*, (2) *Infinitely many solutions* or (3) *A unique solution*. Display all solutions found.

$$\begin{array}{rcrcl} x & + & ky & = & 2, \\ (2-k)x & + & y & = & 3. \end{array}$$

The solution of this problem involves construction of perhaps three frame sequences, the last frame of each resulting in one of the three possibilities (1), (2), (3).

A portion of the frame sequence is constructed, as follows.

$$\boxed{\begin{array}{rcrcr} x & + & ky & = & 2, \\ (2-k)x & + & y & = & 3. \end{array}}$$

Frame 1.

Original system.

$$\boxed{\begin{array}{rcrcr} x & + & ky & = & 2, \\ & & [1+k(k-2)]y & = & 2(k-2)+3. \end{array}}$$

Frame 2.

`combo(1,2,k-2)`

$$\boxed{\begin{array}{rcrcr} x & + & ky & = & 2, \\ & & (k-1)^2 y & = & 2k-1. \end{array}}$$

Frame 3.

Simplify.

The three expected frame sequences share these initial frames. At this point, we identify the values of k that split off into the three possibilities.

$$\begin{array}{rcl} x + ky &=& 2, \\ (k-1)^2 y &=& 2k-1. \end{array}$$

Frame 3.

Simplify.

There will be a signal equation if the second equation of Frame 3 has no variables, but the resulting equation is not "$0 = 0$." This happens exactly for $k = 1$. The resulting signal equation is "$0 = 1$." We conclude that one of the three frame sequences terminates with the *no solution case*. This frame sequence corresponds to $k = 1$.

Otherwise, $k \neq 1$. For these values of k, there are zero free variables, which implies a unique solution. A by-product of the analysis is that the *infinitely many solutions* case never occurs!

The conclusion

The initially expected three frame sequences reduce to two frame sequences. One sequence gives no solution and the other sequence gives a unique solution.

The three answers:

> *(1) No solution occurs only for $k = 1$.*
> *(2) Infinitely many solutions occurs for no value of k.*
> *(3) A unique solution occurs for $k \neq 1$.*

$$x = 2 - \frac{k(2k-1)}{(k-1)^2},$$
$$y = \frac{(2k-1)}{(k-1)^2}.$$