Project 8. Solve problems L8.1 to L8.5. The problem headers:

- **PROBLEM L8.1. EARTHQUAKE MODEL FOR A BUILDING.**
- **PROBLEM L8.2. TABLE OF NATURAL FREQUENCIES AND PERIODS.**
- **PROBLEM L8.3. UNDETERMINED COEFFICIENTS STEADY-STATE SOL**
- **PROBLEM L8.4. PRACTICAL RESONANCE.**
- **PROBLEM L8.5. EARTHQUAKE DAMAGE.**

**SIX FLOOR Model.**
Refer to the textbook of Edwards-Penney, section 7.4, page 437.
Consider a building with six floors each weighing 50 tons. Each floor corresponds to a restoring Hooke’s force with constant $k=5$ tons/foot. Assume that ground vibrations from the earthquake are modeled by $(1/4)\cos(\omega t)$ with period $T=2\pi/\omega$.

**PROBLEM L8.1. BUILDING MODEL FOR AN EARTHQUAKE.**
Model the 6-floor problem in Maple.
Define the 6 by 6 mass matrix $M$ and Hooke’s matrix $K$ for this system and convert $M\ddot{x}+Kx$ into the system $\ddot{x}=Ax$ where $A$ is defined by textbook equation (1), page 437.
Sanity check: Mass $m=3125$, and the 6x6 matrix contains fraction 16/5.

Then find the eigenvalues of the matrix $A$ to six digits, using the Maple command "eigenvals(A)."
Sanity check: All six eigenvalues should be negative.

```maple
# Sample Maple code for a model with 4 floors.
# Use maple help to learn about evalf and eigenvals.
# A:=matrix([[ -20,10,0,0], [10,-20,10,0],
# [0,10,-20,10],[0,0,10,-10]]);
# with(linalg): evalf(eigenvals(A));
```

**PROBLEM L8.2. TABLE OF NATURAL FREQUENCIES AND PERIODS.**
Refer to figure 7.4.17, page 437.

Find the natural angular frequencies $\omega=\sqrt{-\lambda}$ for the six story building and also the corresponding periods $2\pi/\omega$, accurate to six digits. Display the answers in a table. Compare with answers in Figure 7.4.17, page 437, for the 7-story case.

```maple
# Sample code for a 4x3 table, 4-story building.
# Use maple help to learn about nops and printf.
```
Problem L8.2

Ev := [-10, -1.206147582, -35.32088886, -23.47296354];
Omega := lambda -> sqrt(-lambda):
format := "%10.6f %10.6f %10.6f\n";
seq(printf(format, ev[i], Omega(ev[i]), 2*evalf(Pi)/Omega(ev[i])),
i = 1..n);

Problem L8.3. Undetermined Coefficients

Steady-State Periodic Solution.

Consider the forced equation \( x' = Ax + \cos(wt)b \) where \( b \) is a constant vector. The earthquake's ground vibration is accounted for by the extra term \( \cos(wt)b \), which has period \( T = \frac{2\pi}{w} \).

The solution \( x(t) \) is the 6-vector of excursions from equilibrium of the corresponding 6 floors.

Sought here is not the general solution, which certainly contains transient terms, but rather the steady-state periodic solution, which is known from the theory to have the form \( x(t) = \cos(wt)c \) for some vector \( c \) that depends only on \( A \) and \( b \).

Define \( b := 0.25*w^2*vector([1, 1, 1, 1, 1, 1]) \): in Maple and find the vector \( c \) in the undetermined coefficients solution \( x(t) = \cos(wt)c \).

Vector \( c \) depends on \( w \). As outlined in the textbook, vector \( c \) can be found by solving the linear algebra problem \(-w^2 c = Ac + b\); see page 433. Don't print \( c \), as it is too complex; instead, print \( c[1] \) as an illustration.

Sample code for defining \( b \) and \( A \), then solving for \( c \) in the 4-floor case.

# See maple help to learn about vector and linsolve.
# w := 'w': u := w*w: b := 0.25*u*vector([1, 1, 1, 1, 1, 1]):
# A := matrix([[ -20, 10, 0, 0, 0, 0], [10, -20, 10, 0, 0, 0], [0, 10, -20, 10, 0, 0], [0, 0, 10, -10, 0, 0]]):
# Au := evalm(A + u*diag(1, 1, 1, 1, 1, 1));
# c := linsolve(Au, -b):
# evalf(c[1], 2);

Problem L8.4. Practical Resonance.

Consider the forced equation \( x' = Ax + \cos(wt)b \) of L8.3 above with \( b := 0.25*w^2*vector([1, 1, 1, 1, 1, 1]) \).

Practical resonance can occur if a component of \( x(t) \) has large amplitude compared to the vector norm of \( b \). For example, an earthquake might cause a small 3-inch excursion on level ground, but the building's floors might have 50-inch excursions, enough to destroy
the building.

Let $\text{Max}(c)$ denote the maximum modulus of the components of vector $c$. Plot $g(T)=\text{Max}(c(u))$ with $w=(2*\Pi)/T$ for periods $T=0$ to $T=6$, ordinates $\text{Max}=0$ to $\text{Max}=10$, the vector $c(w)$ being the answer produced in L8.3 above. Compare your figure to the textbook Figure 7.4.18, page 438.

```
# Sample maple code to define the function Max(c), 4-floor building.
# Use maple help to learn about norm, vector, subs and linsolve.
# with(linalg):
# w:='w': Max:= c -> norm(c, infinity); u:=w*w:
# b:=0.25*w*w*vector([1,1,1,1]):
# A:=matrix([-20,10,0,0], [10,-20,10,0], [0,10,-20,10], [0,0,10,-10]));
# Au:=evalm(A+u*diag(1,1,1,1));
# C:=ww -> subs(w=ww,linsolve(Au,-b)):
# plot(Max(C(2*Pi/r)),r=0..6,0..10,numpoints=150);
```

**PROBLEM L8.4. WARNING: Save your file often!!!**

(a) Re-plot the amplitudes in L8.4 for periods 1.5 to 5.5 and amplitudes 5 to 10.
There will be five spikes.
(b) Create five zoom-in plots, one for each spike, choosing a
$T$-interval that shows the full spike.
(c) Determine from the five zoom-in plots approximate intervals for
the period $T$ such that some floor in the building will undergo
excursions from equilibrium in excess of 5 feet.

```
# Example: Zoom-in on a spike for amplitudes 5 feet to 10 feet,
# periods 1.97 to 2.01.
#with(linalg): w:='w': Max:= c -> norm(c, infinity); u:=w*w:
#Au:=matrix([-20+u,10,0,0], [10,-20+u,10,0],
#[0,10,-20+u,10],[0,0,10,-10+u]);
##b:=0.25*w*w*vector([1,1,1,1]):
#C:=ww -> subs(w=ww,linsolve(Au,-b)):
#plot(Max(C(2*Pi/r)),r=1.97..2,01,5..10,numpoints=150);
```

**PROBLEM L8.5. EARTHQUAKE DAMAGE.**

The maximum amplitude plot of L8.4 can be used to detect the
ground vibration of period $T$. A ground vibration $(1/4)\cos(\omega t)$,
$T=2*\Pi/\omega$, will be assumed, as in L8.4.

(a) Replot the amplitudes in L8.4 for periods 1.5 to 5.5 and
amplitudes 5 to 10.
There will be five spikes.
(b) Create five zoom-in plots, one for each spike, choosing a
$T$-interval that shows the full spike.
(c) Determine from the five zoom-in plots approximate intervals for
the period $T$ such that some floor in the building will undergo
excursions from equilibrium in excess of 5 feet.

```
# Example: Zoom-in on a spike for amplitudes 5 feet to 10 feet,
#periods 1.97 to 2.01.
#with(linalg): w:='w': Max:= c -> norm(c, infinity); u:=w*w:
#Au:=matrix([-20+u,10,0,0], [10,-20+u,10,0],
#[0,10,-20+u,10],[0,0,10,-10+u]);
##b:=0.25*w*w*vector([1,1,1,1]):
#C:=ww -> subs(w=ww,linsolve(Au,-b)):
#plot(Max(C(2*Pi/r)),r=1.97..2,01,5..10,numpoints=150);
```

**PROBLEM L8.5. WARNING: Save your file often!!**

(a) Re-plot the five spikes.
# plot(Max(C(2*Pi/r)),r=1.97..2,01,5..10,numpoints=150);
# (b) Plot five zoom-in graphs.
# one:=1.79..1.83:plot(Max(C(2*Pi/r)),r=one,5..10,numpoints=150);
# two:=???:plot(Max(C(2*Pi/r)),r=two,5..10,numpoints=150);
# three:=???:plot(Max(C(2*Pi/r)),r=three,5..10,numpoints=150);
# four:=???:plot(Max(C(2*Pi/r)),r=four,5..10,numpoints=150);
# five:=???:plot(Max(C(2*Pi/r)),r=five,5..10,numpoints=150);
# (c) Print period ranges.
# PeriodRanges:=[one,two,three,four,five];