

**Math 2250 Extra Credit Problems**  
**Chapter 9**  
**Fall 2009**

**Due date:** See the internet due date for 9.4, which is the due date for these problems. Records are locked on that date and only corrected, never appended.

**Submitted work.** Please submit one stapled package per problem. Kindly label problems Extra Credit. Label each problem with its corresponding problem number, e.g., Xc9.1-4. You may attach this printed sheet to simplify your work.

**Problem Xc9.1-4. (Phase Portraits)**

Find the equilibrium points for the system. Plot a phase diagram using the maple code below.

$$\begin{cases} \frac{dx}{dt} = x - 2y + 3, \\ \frac{dy}{dt} = x - y + 2. \end{cases}$$

**Example:** Plot the phase diagram of  $\mathbf{u}' = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix} \mathbf{u} + \begin{pmatrix} 4 \\ 5 \end{pmatrix}$  using maple.

```
with(DEtools):
equilEQ:=[0=x+2*y+4,0=3*y+5];
solve(equilEQ,{x,y});# find diagram center (a,b)
a:=-2/3;b:=-5/3;
de:=[diff(x(t),t)=x(t)+2*y(t)+4,diff(y(t),t)=3*y(t)+5];
ic:=[[x(0)=0,y(0)=-1],[x(0)=-1,y(0)=-1.5],[x(0)=0.5,y(0)=-2],
[x(0)=0.5,y(0)=-1.5],[x(0)=-0.7,y(0)=-1.7]];
DEplot(de,[x(t),y(t)],t=-10..10,ic,x=a-2..a+2,y=b-2..b+2,stepsize=0.05);
```

**Problem Xc9.1-8. (Equilibrium Points)**

Find the equilibrium points for the system. Plot a phase diagram. The graph window should include the three equilibrium points.

$$\begin{cases} \frac{dx}{dt} = x - 2y, \\ \frac{dy}{dt} = 4x - x^3. \end{cases}$$

**Problem Xc9.1-18. (Stability)**

Determine if the equilibrium point  $(0, 0)$  is stable, asymptotically stable, or unstable. Identify the equilibrium point as a node, saddle, center or spiral by examination of its computer-generated direction field.

- (a)  $x' = y, y' = -x$
- (b)  $x' = y, y' = -5x - 4y$
- (c)  $x' = -2x, y' = -2y$
- (d)  $x' = y, y' = x$

**Problem Xc9.2-2. (Classification by Eigenvalues)**

Compute the eigenvalues of  $A$ . Determine stability of equilibrium  $(0, 0)$  and classify as node (proper/improper), saddle, spiral, center.

(a)  $A \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

(b)  $A \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$

(c)  $A \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix}$

(c)  $A \begin{pmatrix} 1 & -2 \\ 2 & -3 \end{pmatrix}$

**Problem Xc9.2-12. (Phase Portrait)**

Find the equilibrium point (it is unique) and plot by computer a phase diagram.

$$\begin{cases} \frac{dx}{dt} = x + y - 7, \\ \frac{dy}{dt} = 3x - y - 5. \end{cases}$$

**Problem Xc9.2-22. (Almost Linear System)**

Linearize the system at its equilibria and determine the stability and type of each. Plot a phase diagram by computer to verify the claims made.

$$\begin{cases} \frac{dx}{dt} = 2x - 5y + x^3, \\ \frac{dy}{dt} = 4x - 6y + y^4. \end{cases}$$

**Problem Xc9.3-8. (Predator-Prey System)**

Linearize the system at equilibrium point  $(0, 0)$ . Verify that the phase diagram of the nonlinear system at  $(0, 0)$  is a saddle.

$$\begin{cases} \frac{dx}{dt} = x(5 - x - y), \\ \frac{dy}{dt} = y(-2 + x). \end{cases}$$

**Problem Xc9.3-9. (Predator-Prey System)**

Linearize the system at equilibrium point  $(5, 0)$ . Verify that the phase diagram of the nonlinear system at  $(5, 0)$  is a saddle.

$$\begin{cases} \frac{dx}{dt} = x(5 - x - y), \\ \frac{dy}{dt} = y(-2 + x). \end{cases}$$

**Problem Xc9.3-10. (Predator-Prey System)**

Linearize the system at equilibrium point  $(2, 3)$ . Verify that the phase diagram of the nonlinear system at  $(2, 3)$  is an asymptotically stable spiral.

$$\begin{cases} \frac{dx}{dt} = x(5 - x - y), \\ \frac{dy}{dt} = y(-2 + x). \end{cases}$$

**Problem Xc9.4-4. (Almost Linear System)**

Linearize at  $(0, 0)$  and classify the equilibrium point  $(0, 0)$  of the nonlinear system, using a phase diagram to verify the conclusion.

$$\begin{cases} \frac{dx}{dt} = 2 \sin x + \sin y, \\ \frac{dy}{dt} = \sin x + 2 \sin y. \end{cases}$$

**Problem Xc9.4-8. (Almost Linear System)**

Linearize at all equilibria and classify the equilibrium points of the nonlinear system. Use a phase diagram to verify the conclusions.

$$\begin{cases} \frac{dx}{dt} = y, \\ \frac{dy}{dt} = \sin \pi x - y. \end{cases}$$

**End of extra credit problems chapter 9.**