Name ____________________________ Class Time _____

Math 2250 Extra Credit Problems
Chapter 6
Fall 2009

Due date: See the internet due date for 8.1, which is the due date for these problems. Records are locked on that date and only corrected, never appended.

Submitted work. Please submit one stapled package per problem. Kindly label problems $\boxed{\textbf{Extra Credit}}$. Label each problem with its corresponding problem number, e.g., $\boxed{\text{Xc6.1-36}}$. You may attach this printed sheet to simplify your work.

Problem Xc6.1-12. (Eigenpairs of a 2×2)

Let $A = \begin{pmatrix} 9 & -10 \\ 2 & 0 \end{pmatrix}$. Find the eigenpairs of A. Then report eigenpair packages P and D such that $AP = PD$.

Problem Xc6.1-20. (Eigenpairs of a 3×3)

Let $A = \begin{pmatrix} 5 & -6 & 3 \\ 6 & -7 & 3 \\ 6 & -6 & 2 \end{pmatrix}$. Find the eigenpairs of A. Then report eigenpair packages P and D such that $AP = PD$.

Problem Xc6.1-32. (Complex eigenpairs of a 2×2)

Let $A = \begin{pmatrix} 0 & -6 \\ 24 & 0 \end{pmatrix}$. Find the eigenpairs of A. Then report eigenpair packages P and D such that $AP = PD$.

Problem Xc6.1-36. (Eigenvalues of band matrices)

Find the eigenvalues of the matrix A below without the aid of computers.

$$A = \begin{pmatrix} 1 & 2 & 0 & 0 & 0 & 0 \\ 2 & 1 & 2 & 0 & 0 & 0 \\ 0 & 2 & 1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 1 & 2 & 0 \\ 0 & 0 & 0 & 2 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Problem Xc6.2-6. (Eigenpair packages of a 3×3)

Let $A = \begin{pmatrix} 2 & -2 & 1 \\ 2 & -2 & 1 \\ 2 & -2 & 1 \end{pmatrix}$. Find the eigenpairs of A. Then report eigenpair packages P and D such that $AP = PD$.
Check the answer by hand, expanding both products AP and PD, finally showing equality.

Problem Xc6.2-18. (Fourier's model for a 3×3)

Assume Fourier's model for a certain matrix A:

$$A\left(c_1\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} + c_2\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + c_3\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right) = 3c_1\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} + c_2\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + c_3\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

Find A explicitly from $AP = PD$. Check your answer by finding the eigenpairs of A.

Problem Xc6.2-28. (Eigenpairs and diagonalization of a 4×4)

Determine the eigenpairs of A below. If diagonalizable, then report eigenpair packages P and D such that $AP = PD$.

$$A = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 1 & 2 & 0 \\ 0 & 2 & 1 & 2 \\ 0 & 0 & 0 & 13 \end{pmatrix}$$

End of extra credit problems chapter 6.