Mathematics 5410
Picard Iteration

Definition. The *Picard iterates* for the problem

$$y' = f(t, y), \quad y(0) = A$$

are defined by the formulas

$$y_0(x) = A,$$
$$y_n(x) = A + \int_0^x f(t, y_{n-1}(t))dt, \quad n = 1, 2, 3, \ldots$$

Example. Find and plot the Picard iterates y_0, y_1, y_2 for the problem

$$y' = y^2, \quad y(0) = 1.$$

Solution: The exact solution is $y = 1/(1-t)$, defined on the interval $0 \le t < 1$. The `Maple 6` code which does the plot appears below.

```
with(plots):
y0:=1:
T:=(f,x)->y0+eval(int(f(t)^2,t=0..x)):
n:=2:
y:=array(0..n): Y:=array(0..n):
y[0]:=x->y0:
for i from 1 to n do
  y[i]:=unapply(T(y[i-1],x),x):
  Y[i]:=plot(y[i](x),x=0..1):
od:
display([seq(Y[i],i=1..n)]);
seq(eval(y[i]),i=1..n);
```

Problem 1. Find and plot the Picard iterates y_0 through y_6 for the problem

$$y' = y^2, \quad y(0) = 5.$$

Compare graphically the convergence of the sequence $\{y_n\}$ to the limit solution $y = 5/(1-5t)$ and discuss the reason for the finite escape time of $t = 1/5$.

Problem 2. Find and plot the Picard iterates y_0 through y_5 for the problem

$$y' = y^4, \quad y(0) = 1.$$

Compare graphically the convergence of the sequence $\{y_n\}$ to the limit solution $y = 1/(1-3t)^{1/3}$ and discuss the reason for the finite escape time of $t = 1/3$.