Differential Equation Plots
Mathematics 5410

Example. Solve the problem

$$\begin{cases} y' = \dfrac{y+1}{t+1} \\ y(1) = 1 \end{cases}$$

numerically and plot.

Solution: In `Maple`, the code is as follows.

```
de:=diff(y(t),t)=(y(t)+1)/(t+1):
ic:=y(1)=1:
Y:=dsolve({de,ic},y(t),type=numeric):
with(plots):
odeplot(Y,[t,y(t)],1..10);
# True solution is y(t)=t.
# Plot should be the 45 degree line y=x.
```

The xmaple command `plotsetup(x11):` redirects plots to a separate window, which makes them somewhat easier to manipulate. The idea is invaluable for seeing a sequence of plots with different initial data. These plots go away by pressing key **Q** inside the plot.

Problem 1. Solve the problem

$$\begin{cases} y' = \dfrac{y^2}{t} \\ y(1) = 1 \end{cases}$$

numerically and plot on $t = 1$ to $t = 5$.

Problem 2. Solve the problem

$$\begin{cases} y' = ye^{-t} \\ y(0) = e \end{cases}$$

numerically and plot on $t = 0$ to $t = 2$.

Problem 3. Solve the problem

$$\begin{cases} y' = -\dfrac{t}{y} \\ y(1) = 2 \end{cases}$$

numerically and plot on $t = 1$ to $t = 5$.

Problem 4. Plot the solution curves for the problem

$$\begin{cases} y' = 3y(1 - y/12) - 8 \\ y(0) = y_0 \end{cases}$$

on $t = 0$ to $t = 1$ for the five cases $y_0 = 2, 4, 6, 8, 10$, and highlight the equilibrium levels.

Example. Plot the solutions to the first two differential equations of Problems 4 on the same plot.

Solution:

```
de:=diff(y(t),t) = 3*y(t)*(1-y(t)/12)-8:
ic:=y(0)=2:
Y1:=dsolve({de,ic},y(t),type=numeric):
p1:=odeplot(Y1,[t,y(t)],0..1):
de:=diff(y(t),t) = 3*y(t)*(1-y(t)/12)-8:
ic:=y(0)=4:
Y2:=dsolve({de,ic},y(t),type=numeric):
p2:=odeplot(Y2,[t,y(t)],0..1):
display([p1,p2]);
```

Problem 5 (Teddy Bears, page 58). Plot the solution curves for the problem

$$\begin{cases} x' = \sin y - 2\sin x^2 \sin 2y, \\ y' = -\cos x - 2x\cos x^2 \cos 2y, , \\ x(0) = 0, \quad y(0) = \pi/2 \end{cases}$$

on $t = -10$ to $t = 10$. Reproduce a portion of the figure on page 59.

Sample code: The code below produces a direction field and a portion of the desired plot.

```
de1:=diff(x(t),t)=sin(y(t))-2*sin(x(t)*x(t))*sin(2*y(t)):
de2:=diff(y(t),t)=-cos(x(t))-2*x(t)*cos(x(t)*x(t))*cos(2*y(t)):
ic:=x(0)=0,y(0)=Pi/2:
with(DEtools):
DEplot({de1,de2},{x(t),y(t)},t=-5..5,[[ic]]);
```