Mathematics 5410
DEplot Revision

Example. Solve the antihistamine problem

$$\begin{cases} x' = F(t) - k_1 x, \\ y' = k_1 x - k_2 y, \\ x(0) = 0, \quad y(0) = 0 \end{cases}$$

numerically for

$$k_1 = 0.6931, \quad k_2 = 0.0462$$

and

$$F(t) = 12 \sum_{j=0}^{20} \left(H(t - 6j) - H(t - 6j - 0.5) \right).$$

Plot the graphic of $y(t)$ on the window $0 \le t \le 120$, $0 \le y \le 60$. See Borrelli–Coleman, page 72.

Solution: In `Maple`, the code to find and plot $y(t)$ is as follows.

```
H:=x->piecewise(x<0,0,1):
F:=x->12*sum(H(x-6*jj)-H(x-6*jj-0.5),jj=0..20):
k1:=0.6931:k2:=0.0462:
de:=diff(x(t),t)=F(t)-k1*x(t),
    diff(y(t),t)=k1*x(t)-k2*y(t):
ic:=[x(0)=0,y(0)=0]:
with(DEtools):
DEplot([de],[x(t),y(t)],t=0..120,[ic],scene=[t,y],stepsize=0.05);
```

Problem 1. Solve the problem

$$\begin{cases} x' = F(t) - k_1 x, \\ y' = k_1 x - k_2 y, \\ x(0) = 0, \quad y(0) = 0 \end{cases}$$

numerically for

$$k_1 = 0.6931, \quad k_2 = 0.0462, 0.0231, 0.0077$$

and

$$F(t) = 12 \sum_{j=0}^{20} \left(H(t - 6j) - H(t - 6j - 0.5) \right).$$

Plot the three graphics of $y(t)$ on a single plot over $0 \le t \le 120$, $0 \le y \le 60$. See Borrelli–Coleman, page 72.

Example. Solve the finite escape time problem for

$$\begin{cases} y' = 4y^2, \\ y(0) = 1 \end{cases}$$

and justify the escape time $T = 1/4$ mathematically.

Solution: In `Maple`, the code to find and plot $y(t)$ is as follows.

```
de:=diff(y(t),t)=4*y(t)*y(t):
ic:=[y(0)=1]:
with(DEtools):
DEplot([de],[y(t)],t=0..0.25,[ic],stepsize=0.05);
```

The argument to find $T = 1/4$ is separation of variables. The solution from this method is $y = 1/(1 - 4t)$.

Problem 2. Solve the problem

$$\begin{cases} y' = y^2, \\ y(0) = 1 \end{cases}$$

numerically and plot to determine the finite escape time $T = 1$. Justify $T = 1$ mathematically by a separation of variables argument and submit this mathematical justification with the computer program.

Problem 3. Solve the problem

$$\begin{cases} y' = 0.1(y-3)(y-1)(y+1), \\ y(0) = y_0 \end{cases}$$

numerically for $y_0 = -3$ to 5 in steps of 0.5. Plot the sixteen solutions on the window $-5 \le x \le 5$, $-3 \le y \le 5$.