Name. ________________________ **Sample Midterm 3, 5410**

Differential Equations 5410

Sample Midterm Exam 3
Tuesday, 9 December 2008

Instructions: This in-class exam is 50 minutes. No calculators, notes, tables or books. No answer check is expected.

1. **(Variation of parameters)**
   (a) [50%] State and prove the variation of parameters formula for a first-order linear vector-matrix differential equation $\mathbf{u}'(t) = A(t)\mathbf{u}(t) + \mathbf{F}(t)$.
   (b) [50%] Write the formula for y_p according to variation of parameters for $y'' - 4y = 4e^{2x}$. **Don't integrate**!

2. (Stability and Phase Portraits) Choose two problems:

(a) [50%] Classify the equilibria as stable or unstable: $x' = 3x - 3y$, $y' = x(y-1)$, $z' = xy - 2yz$.

(b) [50%] Prove that a constant-coefficient equation $x''' + a_2x'' + a_1x' + a_0x = 0$ is asymptotically stable at $x = 0$, if the real part of each root of the characteristic equation is negative.

(c) [50%] Classify as a stable or unstable center, spiral, saddle or node: $\mathbf{x}' = A\mathbf{x}$, for the three matrices

(c-1) $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$,

(c-2) $A = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$,

(c-3) $A = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$.

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3. (Linear 3×3 systems)

(a) [50%] For $x' = 2x$, $y' = -x + y$, $z' = -x + z$, give Putzer's spectral recipe answer, without simplifications (don't multiply out factors $A - \lambda I$).

(b) [50%] Solve $x' = 2x$, $y' = x + y$, $z' = -x + z$ by eigenanalysis.

4. (Resonance)

(a) [50%] Prove that the amplitude of the unique periodic solution of $mx'' + cx' + kx = F_0 \cos(\omega t)$ is monotonic in the variable $c > 0$.

(b) [50%] Why is the practical resonance frequency for $mx'' + cx' + kx = F_0 \cos(\omega t)$ always less than the pure resonance frequency for $mx'' + kx = F_0 \cos(\omega t)$?

5. (Theory of linear systems)

(a) [50%] Given $ty'' + (1-t)y' + (\tan t)y = e^t \cos t$, find the possible maximal intervals of existence of a solution $y(t)$.

(b) [25%] State the superposition principle for $\mathbf{x}' = A(t)\mathbf{x}$.

(c) [25%] In the proof of Abel's formula for $\mathbf{x}' = A(t)\mathbf{x}$, in the $n \times n$ case, some properties of determinants are applied. State the two most important properties.