Mathematics 5410
Picard-Lindelof Project

The Project: State and prove the Picard-Lindelof theorem for existence and uniqueness of the initial value problem $y' = f(t, y)$, $y(t_0) = y_0$, subject to the five ground rules below.

1. Assume f and y are scalar functions.

2. Assume f satisfies a local Lipschitz condition and f is continuous.

3. You may use the outline of proof found in Kreider-Ostberg Chapter 9, or the one found in Edwards-Penney 2nd edition, Appendix A. The two outlines assume different mathematical backgounds.

   - The proof in Kreider-Ostberg applies the Banach contraction mapping theorem. In particular, do not repeat material found in Kreider-Ostberg Sections 9.1, 9.2, 9.3. Simply cite the results as needed. A xerox of essential pages will be provided. There are a number of gaps in the proof, missing details from mathematical analysis, and in general a lot of reference look-up to complete the proof. Don Kreider died in December 2006, so there will not be another edition of the book.
   - The proof in Edwards-Penney assumes an advanced calculus background only, which must include the theory of uniform convergence, sequences of functions and limits of sequences. The appendix divides the existence and uniqueness into two proofs, which use vector notation, but it is easy to remove that notation and focus on the scalar case. This path requires finding references from Taylor-Mann and filling in the gaps in the presentation.

4. Make the proof brief and easy to read.

5. Submit by 15 November 2008.