

**Math 2270 Extra Credit Problems**  
**Chapter 1**  
**August 2008**

**Due date:** See the internet due dates. Records are locked on that date and only corrected, never appended.

**Submitted work.** Please submit one stapled package per problem. Kindly label problems Extra Credit. Label each problem with its corresponding problem number. You may attach this printed sheet to simplify your work.

**Problem XC1.1-26. (Three possibilities)**

Determine which values of  $k$  correspond to (a) a unique solution, (b) no solution or (c) infinitely many solutions.

$$\begin{cases} x + 2y + kz = 0 \\ 2x + 4y + (k+1)z = 2 \\ 3x + 6y + (2k+1)z = 2 \end{cases}$$

**Problem XC1.1-30. (Polynomial interpolation)**

Find the polynomial  $f(x) = a + bx + cx^2$  which passes through the points  $(1, 9)$ ,  $(2, 24)$ ,  $(3, 47)$ .

**Problem XC1.1-32. (Polynomial interpolation)**

Find all polynomials  $f(x) = a + bx + cx^2$  which pass through the points  $(1, 10)$ ,  $(2, 28)$  and  $f'(3) = 33$ .

**Problem XC1.2-22. (RREF)**

Report five types of  $3 \times 4$  matrices in RREF form.

**Problem XC1.2-28. (Combo rule)**

Consider the following systems.

$$(1) \quad \begin{cases} a_{11}x_1 + \cdots + a_{1n}x_n = b_1 \\ \vdots \\ a_{m1}x_1 + \cdots + a_{mn}x_n = b_m \end{cases}$$

$$(2) \quad \begin{cases} a_{11}x_1 + \cdots + a_{1n}x_n = b_1 \\ \vdots \\ c_{k1}x_1 + \cdots + c_{kn}x_n = d_k \\ \vdots \\ a_{m1}x_1 + \cdots + a_{mn}x_n = b_m \end{cases}$$

We assume the systems identical except for equation  $k$ , which is obtained in the second system by applying a combination rule  $\text{combo}(\mathbf{r}, \mathbf{k}, \mathbf{c})$  to the first system. We assume  $r \neq k$  and then the coefficients in the second system are given by

$$c_{kj} = a_{kj} + ca_{rj}, \quad j = 1, \dots, n, \quad d_k = b_k + cb_r.$$

(a) Prove that every solution of system (1) is a solution of system (2).

(b) Prove that every solution of system (2) is a solution of system (1).

**Problem XC1.2-30. (Polynomial interpolation)**

Find the polynomial  $f(x) = a + bx + cx^2 + dx^3$  which satisfies  $f(1) = 8$ ,  $f(2) = 24$ ,  $f(3) = 24$ ,  $f(4) = 110$ .

**Problem XC1.3-26. (Matrix algebra)**

Find a  $3 \times 3$  matrix  $A$  which satisfies the following relations.

$$A \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad A \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad A \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

**End of extra credit problems chapter 1.**