Name ______________________ Class Time _____

Math 2250 Extra Credit Problems
Chapter 9
December 2008

Due date: See the internet due date for 9.4, which is the due date for these problems. Records are locked on that date and only corrected, never appended.

Submitted work. Please submit one stapled package per problem. Kindly label problems **Extra Credit**. Label each problem with its corresponding problem number, e.g., Xc9.1-4. You may attach this printed sheet to simplify your work.

Problem Xc9.1-4. (Phase Portraits)

Find the equilibrium points for the system. Plot a phase diagram using the `maple` code below.

$$\begin{cases} \dfrac{dx}{dt} &= x - 2y + 3, \\ \dfrac{dy}{dt} &= x - y + 2. \end{cases}$$

Example: Plot the phase diagram of $\mathbf{u}' = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix} \mathbf{u} + \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ using `maple`.

```
with(DEtools):
equilEQ:=[0=x+2*y+4,0=3*y+5];
solve(equilEQ,{x,y});# find diagram center (a,b)
a:=-2/3;b:=-5/3;
de:=[diff(x(t),t)=x(t)+2*y(t)+4,diff(y(t),t)=3*y(t)+5];
ic:=[[x(0)=0,y(0)=-1],[x(0)=-1,y(0)=-1.5],[x(0)=0.5,y(0)=-2],
[x(0)=0.5,y(0)=-1.5],[x(0)=-0.7,y(0)=-1.7]];
DEplot(de,[x(t),y(t)],t=-10..10,ic,x=a-2..a+2,y=b-2..b+2,stepsize=0.05);
```

Problem Xc9.1-8. (Equilibrium Points)

Find the equilibrium points for the system. Plot a phase diagram. The graph window should include the three equilibrium points.

$$\begin{cases} \dfrac{dx}{dt} &= x - 2y, \\ \dfrac{dy}{dt} &= 4x - x^3. \end{cases}$$

Problem Xc9.1-18. (Stability)

Determine if the equilibrium point $(0, 0)$ is stable, asymptotically stable, or unstable. Identify the equilibrium point as a node, saddle, center or spiral by examination of its computer-generated direction field.

(a) $x' = y$, $y' = -x$

(b) $x' = y$, $y' = -5x - 4y$

(c) $x = -2x$, $y' = -2y$

(d) $x = y$, $y' = x$

Problem Xc9.2-2. (Classification by Eigenvalues)

Compute the eigenvalues of A. Determine stability of equilibrium $(0, 0)$ and classify as node (proper/improper), saddle, spiral, center.

(a) $A\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

(b) $A\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$

(c) $A\begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix}$

(c) $A\begin{pmatrix} 1 & -2 \\ 2 & -3 \end{pmatrix}$

Problem Xc9.2-12. **(Phase Portrait)**

Find the equilibrium point (it is unique) and plot by computer a phase diagram.

$$\begin{cases} \dfrac{dx}{dt} &= x + y - 7, \\ \dfrac{dy}{dt} &= 3x - y - 5. \end{cases}$$

Problem Xc9.2-22. **(Almost Linear System)**

Linearize the system at its equilibria and determine the stability and type of each. Plot a phase diagram by computer to verify the claims made.

$$\begin{cases} \dfrac{dx}{dt} &= 2x - 5y + x^3, \\ \dfrac{dy}{dt} &= 4x - 6y + y^4. \end{cases}$$

Problem Xc9.3-8. **(Predator-Prey System)**

Linearize the system at equilibrium point $(0, 0)$. Verify that the phase diagram of the nonlinear system at $(0, 0)$ is a saddle.

$$\begin{cases} \dfrac{dx}{dt} &= x(5 - x - y), \\ \dfrac{dy}{dt} &= y(-2 + x). \end{cases}$$

Problem Xc9.3-9. **(Predator-Prey System)**

Linearize the system at equilibrium point $(5, 0)$. Verify that the phase diagram of the nonlinear system at $(5, 0)$ is a saddle.

$$\begin{cases} \dfrac{dx}{dt} &= x(5 - x - y), \\ \dfrac{dy}{dt} &= y(-2 + x). \end{cases}$$

Problem Xc9.3-10. **(Predator-Prey System)**

Linearize the system at equilibrium point $(2, 3)$. Verify that the phase diagram of the nonlinear system at $(2, 3)$ is an asymptotically stable spiral.

$$\begin{cases} \dfrac{dx}{dt} &= x(5 - x - y), \\ \dfrac{dy}{dt} &= y(-2 + x). \end{cases}$$

Problem Xc9.4-4. **(Almost Linear System)**

Linearize at $(0,0)$ and classify the equilibrium point $(0,0)$ of the nonlinear system, using a phase diagram to verify the conclusion.

$$\begin{cases} \dfrac{dx}{dt} &= 2\sin x + \sin y, \\ \dfrac{dy}{dt} &= \sin x + 2\sin y. \end{cases}$$

Problem Xc9.4-8. (Almost Linear System)

Linearize at all equilibria and classify the equilibrium points of the nonlinear system. Use a phase diagram to verify the conclusions.

$$\begin{cases} \dfrac{dx}{dt} &= y, \\ \dfrac{dy}{dt} &= \sin \pi x - y. \end{cases}$$

End of extra credit problems chapter 9.