Name ____________________ Class Time _____

Math 2250 Extra Credit Problems
Chapter 4
August 2008

Due date: See the internet due date for 6.1, which is the due date for these problems. Records are locked on that date and only corrected, never appended.

Submitted work. Please submit one stapled package per problem. Kindly label problems **Extra Credit**. Label each problem with its corresponding problem number, e.g., Xc4.1-20. You may attach this printed sheet to simplify your work.

Problem XcL3.1. (maple lab 3)

You may submit this problem only for score increases on maple lab 3.

Solve symbolically by chapter 1 methods the initial value problem $y' = 2xy^2$, $y(0) = 1$. Do an answer check in maple or by hand. Answer: $y = 1/(1 - x^2)$. This problem has no numerical work!

Problem XcL3.2. (maple lab 3)

You may submit this problem only for score increases on maple lab 3. This problem counts as three (3) problems.

Solve $y' = 2xy^2$, $y(0) = 1$ numerically for the value of $y(0.5)$ using (1) Euler's method, (2) Heun's method and (3) the RK4 method. Use step size $h = 0.1$. Include computer code and a print of the data. Report the answers in a table for x-values 0, 0.1, 0.2, 0.3, $0, 4$, 0.5.

Problem XcL4.1. (maple lab 4)

You may submit this problem only for score increases on maple lab 4.

Solve symbolically by chapter 1 methods the initial value problem $y' = e^{-y}$, $y(0) = 0$. Do an answer check in maple or by hand. Answer: $y = \ln(1 + x)$. This problem has no numerical work!

Problem XcL4.2. (maple lab 4)

You may submit this problem only for score increases on maple lab 4. This problem counts as three (3) problems.

Solve $y' = e^{-y}$, $y(0) = 0$ numerically for the value of $y(1.0)$ using (1) Euler's method, (2) Heun's method and (3) the RK4 method. Use step size $h = 0.001$. Include a computer code appendix in the report, but do not print the data. Report the answers in a table for x-values 0, 0.2, $0, 4$, 0.6, 0.8, 1.0. Include the percentage error $E = 100|\ln(2) - y(1.0)|/|\ln(2)|$ in your report, one error report for each of the three methods.

Problem Xc4.1-20. (Independence)

Test independence or dependence: $\begin{pmatrix} 5 \\ 5 \\ 4 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$.

Problem Xc4.1-30. (Kernel theorem)

Verify from the kernel theorem (Theorem 2, 4.2) that the set of all vectors in $\mathcal{R}^3$ such that $2x - y = 3z$ is a subspace of $\mathcal{R}^3$.

Problem Xc4.1-32. (Subspace criterion)

Apply the subspace criterion (Theorem 1, 4.2) to verify that the set of all vectors in $\mathcal{R}^3$ such that $2x - y = 3z$, $xy = 0$ is **not** a subspace of $\mathcal{R}^3$.

Problem Xc4.2-4. (Subspace criterion)

Apply the subspace criterion (Theorem 1, 4.2) to verify that the set of all vectors in $\mathcal{R}^3$ satisfying $|x| = y + z$ fails to be a subspace of $\mathcal{R}^3$.

Problem Xc4.2-28. (Kernel theorem)

Let S be the subset of $\mathcal{R}^4$ defined by the equations

$$\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 4 & 0 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

Use the kernel theorem (Theorem 2, 4.2) to prove that S is a subspace of $\mathcal{R}^4$.

Problem Xc4.3-18. (Dependence and frame sequences)

Give the vectors below, display a frame sequence from the augmented matrix C of the vectors to final frame $\mathbf{rref}(C)$. Use the sequence to decide if the vectors are independent or dependent. If dependent, then report all possible dependency relations $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 = \mathbf{0}$.

$$\mathbf{v}_1 = \begin{pmatrix} 3 \\ 9 \\ 0 \\ 5 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} -2 \\ 4 \\ -10 \\ 10 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 4 \\ 7 \\ 5 \\ 0 \end{pmatrix}.$$

Problem Xc4.3-24. (Independence in abstract vector spaces)

Let V be an abstract vector space whose packages of data items $\mathbf{v}$ have unknown details. You are expected to use only definitions and the toolkit of 8 properties in the details of this problem.

Assume given $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ independent vectors in V. Define $\mathbf{u}_1 = \mathbf{v}_1 + 2\mathbf{v}_2$, $\mathbf{u}_2 = \mathbf{v}_3 + \mathbf{u}_1$, $\mathbf{u}_3 = \mathbf{v}_1 + \mathbf{u}_2$. Prove that $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$ are independent.

Problem Xc4.4-6. (Basis)

Find a basis for $\mathcal{R}^3$ which includes two independent vectors $\mathbf{v}_1$, $\mathbf{v}_2$ which are in the plane $2x - 3y + 5z = 0$ and a vector $\mathbf{v}_3$ which is perpendicular to both $\mathbf{v}_1$ and $\mathbf{v}_2$. Hint: review the cross product, a topic from calculus.

Problem Xc4.4-24. (Basis for $A\mathbf{x} = \mathbf{0}$)

Display a frame sequence starting with A having final frame $\mathbf{rref}(A)$. Use this sequence to find the scalar general solution of $A\mathbf{x} = \mathbf{0}$ and then the vector general solution of $A\mathbf{x} = \mathbf{0}$. Finally, report a basis for the solution space of $A\mathbf{x} = \mathbf{0}$.

$$A = \begin{pmatrix} 1 & -4 & -3 & -7 \\ 2 & -1 & 1 & 7 \\ 1 & 2 & 3 & 11 \\ 2 & 6 & 4 & 8 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}.$$

Problem Xc4.5-6. (Row and columns spaces)

Find a basis for the row space and the column space of A, but the bases reported must be rows of A and columns of A, respectively.

$$A = \begin{pmatrix} 1 & -4 & -3 & -7 \\ 2 & -1 & 1 & 7 \\ 2 & 6 & 4 & 8 \end{pmatrix}$$

Problem Xc4.5-24. (Redundant columns)

Use the pivot theorem (Algorithm 2, 4.5) to find the non-pivot columns of A (called redundant columns).

$$A = \begin{pmatrix} 1 & -4 & -3 & -7 & 4 & 3 \\ 2 & -1 & 1 & 7 & 2 & 0 \\ 1 & 2 & 3 & 11 & 0 & 0 \\ 2 & 6 & 4 & 8 & 0 & 1 \end{pmatrix}$$

Problem Xc4.5-28. (Rank and the three properties)

Suppose A is a 5×4 matrix and $A\mathbf{x} = \mathbf{0}$ has a basis of size 3. What are the possible forms of $\mathbf{rref}(A)$?

Problem Xc4.6-2. **(Orthogonality)**

Let A be a 3×3 matrix such that $AA^T = I$. Prove that the columns $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ of A satisfy the orthogonality relations

$$|\mathbf{v}_1| = |\mathbf{v}_2| = \mathbf{v}_3| = 1, \quad \mathbf{v}_1 \cdot \mathbf{v}_2 = \mathbf{v}_2 \cdot \mathbf{v}_3 = \mathbf{v}_3 \cdot \mathbf{v}_1 = 0.$$

Problem Xc4.7-10. **(Subspaces of function spaces)**

Let V be the function space of all polynomials of degree less than 5. Define S to be the subset of V consisting of all polynomials $p(x)$ in V such that $p(0) = p(1)$ and $\int_{-1}^{1} p(x)dx = p(2)$. Prove that S is a subspace of V.

Problem Xc4.7-20. **(Partial fractions and independence)**

Assume the polynomials 1, x, $\ldots$, x^n are *independent*. They are a basis for a vector space V. Use this fact explicitly in the details for determining the constants A, B, C, D in the partial fraction expansion

$$\frac{x^2 - x + 1}{(x-1)(x+1)^2(x-3)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{D}{x-3}.$$

Problem Xc4.7-26. **(Solution space basis for a DE)**

Find the general solution of $3y'' + 5y' = 0$, containing symbols c_1, c_2 for the arbitrary constants. Take partial derivatives $\partial y/\partial c_1$, $\partial y/\partial c_2$ to identify two functions of x. Prove that these functions are independent and hence find a basis for the solution space of the differential equation. Suggestion: View $3y'' + 5y' = 0$ as two equations $3v' + 5v = 0$ and $y' = v$. Then use elementary first order differential equation methods.

End of extra credit problems chapter 4.