Name: [illegible]

Applied Linear Algebra 2270-1
Midterm Exam 1
Wednesday, 26 September 2007

100

Instructions: This in-class exam is designed for 50 minutes. No tables, notes, books or calculators allowed.

1. (Inverse of a matrix) Supply details for two of these:

a. If A and B are $n \times n$ invertible, then $(A+B)^{-1} = A^{-1} + B^{-1}$ can be false.

b. If square matrices A and B satisfy $AB = I$, then $A\mathbf{x} = \mathbf{0}$ cannot have infinitely many solutions.

c. Give an example of a 3×3 matrix A and a frame sequence with three or more frames, starting at A, which proves that $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions.

d. Give an example of a 4×3 system having a unique solution.

d) $\begin{cases} x_1 + 2x_2 + x_3 = 0 \\ x_2 + 3x_3 = 0 \\ 2x_1 + 3x_2 + 9x_3 = 0 \\ 2x_1 + 4x_2 + 2x_3 = 0 \end{cases}$

$$A_1 = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 3 & 0 \\ 2 & 3 & 9 & 0 \\ 2 & 4 & 2 & 0 \end{array}\right]$$

$$A_3 = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & -1 & 7 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \\ \\ \text{combo}(1,3,-2) \\ \text{combo}(1,4,-2) \end{array}$$

$$A_5 = \left[\begin{array}{ccc|c} 1 & 0 & -5 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \text{combo}(2,1,-2) \\ \\ \text{combo}(2,3,1) \\ \\ \end{array}$$

$$A_6 = \left[\begin{array}{ccc|c} 1 & 0 & -5 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \\ \\ \text{mult}(3,1/10) \\ \\ \end{array}$$

See next page ↓

c) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 1 & 2 & 3 \end{bmatrix}$

$A\vec{x} = \vec{0}$

$$\text{aug}(A, \vec{0}) = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 2 & 5 & 4 & 0 \\ 1 & 3 & 1 & 0 \end{array}\right]$$

$$A_1 = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & -2 & 0 \\ 1 & 3 & 1 & 0 \end{array}\right] \begin{array}{l} \\ \text{combo}(1,2,-1) \\ \\ \end{array}$$

$$A_2 = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 1 & -2 & 0 \end{array}\right] \begin{array}{l} \\ \\ \text{combo}(1,3,-1) \end{array}$$

$$A_3 = \left[\begin{array}{ccc|c} 1 & 0 & 7 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 1 & -2 & 0 \end{array}\right] \begin{array}{l} \text{combo}(2,1,-2) \\ \\ \\ \end{array}$$

$$A_4 = \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 7 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array} \begin{array}{l} \\ \\ \\ \text{combo}(2,3,-1) \end{array}$$

x_3 is a free variable, therefore, there are infinitely many solns. for $\vec{x}$.

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1d) (cont'd) $A_8 = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]\begin{array}{l} \text{[illegible]} \\ \text{combo}(3,2,-3) \\ \\ \\ \end{array}$

This system has unique soln.

$x_1 = 0$

$x_2 = 0$

$x_3 = 0$

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2. (Elementary Matrices) Let A be a 3×3 matrix. Let

$$F = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}.$$

Assume F is obtained from A by the following sequential row operations: (1) Swap rows 1 and 3; (2) Add -2 times row 2 to row 3; (3) Add 3 times row 1 to row 2; (4) Multiply row 2 by 5.

a. Write a matrix multiplication formula for F in terms of explicit elementary matrices and the matrix A. (80%)

b. Find A. (20%)

a) $F = E_4 E_3 E_2 E_1 A$

$$E_1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \quad E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A}$$

$$E_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

b) $A = E_1^{-1} E_2^{-1} E_3^{-1} E_4^{-1} F$

$$E_1^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad E_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \quad E_3^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E_4^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 1/5 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ -3 & -3 & -29/5 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ -3 & -3 & -29/5 \\ -6 & -6 & -58/5 \end{bmatrix}$$

$$\boxed{A = \begin{bmatrix} -6 & -6 & -58/5 \\ -3 & -3 & -29/5 \\ 1 & 1 & 2 \end{bmatrix}}$$

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3. **(RREF method)**

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Part I. State a theorem about homogeneous systems which concludes that the $m \times n$ system has at least one nonzero solution. Then supply a proof of the theorem. [20%]

Part II. Let a, b and c denote constants and consider the system of equations

$$\begin{pmatrix} 1 & b-c & a \\ 1 & c & -a \\ 2 & b & a \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -a \\ a \\ a \end{pmatrix}$$

a. Determine those values of a, b and c such that the system has a unique solution. (40%)

b. Determine those values of a, b and c such that the system has no solution. (20%)

c. Determine those values of a, b and c such that the system has infinitely many solutions. (20%)

To save time, don't attempt to solve the equations or to produce a complete frame sequence.

I) Thm: If A is an $m \times n$ matrix, and $\text{rank}(A) < n$, then the system $A\vec{x} = \vec{0}$ has at least 1 nonzero soln.

Proof: $\text{rank}(A) + \text{nullity}(A) = n$ where n = # of variables.

$\text{nullity}(A) = n - \text{rank}(A)$

$\text{nullity}(A) \neq 0$ since $\text{rank}(A) < n$

$\therefore$ There is at least 1 free variable, and thus ∞-many solns.

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see back →

$c-(b-c)$ $\quad$ $b-2(b-c)$

$2c-b$ $\quad$ $b-2b+2c$

Part II)

$$F_1 = \left[\begin{array}{ccc|c} 1 & b-c & a & -a \\ 1 & c & -a & a \\ 2 & b & a & a \end{array}\right]$$

$$F_2 = \left[\begin{array}{ccc|c} 1 & b-c & a & -a \\ 0 & 2c-b & -2a & 2a \\ 2 & b & a & a \end{array}\right] \text{combo}(1,2,-1)$$

$$F_3 = \left[\begin{array}{ccc|c} 1 & b-c & a & -a \\ 0 & 2c-b & -2a & 2a \\ 0 & 2c-b & -a & 3a \end{array}\right] \text{combo}(1,3,-2)$$

$$F_4 = \left[\begin{array}{ccc|c} 1 & b-c & a & -a \\ 0 & 2c-b & -2a & 2a \\ 0 & 0 & a & a \end{array}\right]$$

a) For the system to have a unique soln., $\boxed{\begin{array}{l} a \neq 0 \\ 2c-b \neq 0 \end{array}}$

b) No soln. case: $\boxed{\begin{array}{l} a \neq 0 \\ 2c-b = 0 \end{array}}$

$$F_5 = \left[\begin{array}{ccc|c} 1 & b-c & a & -a \\ 0 & 0 & -2a & 2a \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$F_6 = \left[\begin{array}{ccc|c} 1 & b-c & a & -a \\ 0 & 0 & 0 & 4a \\ 0 & 0 & 1 & 1 \end{array}\right] \rightarrow \text{signal eq.}$$

c) ∞ many soln. case: $\boxed{\begin{array}{l} a = 0 \\ 2c-b = \text{arbitrary} \end{array}}$

$$F_5 = \left[\begin{array}{ccc|c} 1 & b-c & 0 & 0 \\ 0 & 2c-b & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

there is at least 1 free variable. If $2c-b=0$, there will be 2 lead variables.

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4. **(Matrix algebra)**
Do two of these:

a. Find all 2×2 matrices A such that $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} A = A \begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix} + A.$

b. Let A be a 4×3 matrix and B a 3×4 matrix. Explain using matrix algebra and the three possibilities why the 4×4 matrix $C = AB$ cannot start a frame sequence that ends in the identity matrix.

c. For 2×2 matrices A, B, prove that $(A+B)(A-B) = A^2 - B^2$ implies that A and B commute.

a) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} A = A \begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix} + A$

Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\begin{pmatrix} c & d \\ a & b \end{pmatrix} = \begin{pmatrix} 2a & a \\ 2c & c \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\begin{pmatrix} c & d \\ a & b \end{pmatrix} = \begin{pmatrix} 3a & a+b \\ 3c & c+d \end{pmatrix}$

$\begin{pmatrix} c-3a & d-(a+b) \\ a-3c & b-(c+d) \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$$\begin{cases} -3a + c = 0 \\ a - 3c = 0 \\ -a - b + d = 0 \\ b - c - d = 0 \end{cases}$$

$$\left[\begin{array}{cccc|c} -3 & 0 & 1 & 0 & 0 \\ 1 & 0 & -3 & 0 & 0 \\ -1 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 \end{array}\right]$$

$$\begin{bmatrix} 1 & 0 & -3 & 0 \\ -3 & 0 & 1 & 0 \\ -1 & -1 & 0 & 1 \\ 0 & 1 & -1 & -1 \end{bmatrix} \text{swap}(1,2)$$

$$\begin{bmatrix} 1 & 0 & -3 & 0 \\ 0 & 0 & -8 & 0 \\ 0 & -1 & -3 & 1 \\ 0 & 1 & -1 & -1 \end{bmatrix} \begin{matrix} \text{combo}(1,2,3) \\ \text{combo}(1,3,1) \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & -3 & 0 \\ 0 & 0 & -8 & 0 \\ 0 & 1 & 3 & -1 \\ 0 & 1 & -1 & -1 \end{bmatrix} \text{mult}(3,-1)$$

$$\begin{bmatrix} 1 & 0 & -3 & 0 \\ 0 & 0 & -8 & 0 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & -4 & 0 \end{bmatrix} \text{combo}(3,4,-1)$$

$$\begin{bmatrix} 1 & 0 & -3 & 0 \\ 0 & 0 & -8 & 0 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \text{mult}(4,-1/4)$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{matrix} \text{combo}(4,1,3) \\ \text{combo}(4,2,8) \\ \text{combo}(4,3,-3) \end{matrix}$$

see back →

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4a) (cont'd)

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\Rightarrow \begin{cases} a = 0 \\ b = d \\ c = 0 \\ d = t_1 \end{cases} \Rightarrow \begin{matrix} a = 0 \\ b = t_1 \\ c = 0 \\ d = t_1 \end{matrix}$$

$$A = \begin{bmatrix} 0 & t_1 \\ 0 & t_1 \end{bmatrix}$$

$$\boxed{A = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} t_1}$$

) $(A+B)(A-B) = A^2 - B^2$

$$LHS = (A+B)(A-B)$$
$$= A(A-B) + B(A-B)$$
$$= A^2 - AB + BA - B^2$$

This term $= RHS \Leftrightarrow AB = BA$

$\therefore$ A and B must commute in order for $(A+B)(A-B) = A^2 - B^2$ to be true.

Name:

100

5. (Geometry and linear transformations)

Part I. Classify $T(\mathrm{x}) = A\mathrm{x}$ geometrically as scaling, projection onto line L, reflection in line L, pure rotation by angle θ, rotation composed with scaling, horizontal shear, vertical shear. Find an equation for L and define θ where applicable. [60%]

a. $A = \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ $\boxed{\text{c.c. rotation + scaling} \quad \theta = \pi/2,\ r = 2}$

b. $A = \begin{pmatrix} 1 & 5 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$ $\boxed{\text{horizontal shear}}$

c. $A = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} u_1^2 & u_1u_2 \\ u_1u_2 & u_2^2 \end{pmatrix}$ $\boxed{\text{projection onto line } L:\ x = 0}$

d. $A = \frac{1}{4}\begin{pmatrix} 3 & \sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$ $\boxed{\text{projection onto line } L:\ y = \frac{\sqrt{3}}{3} \cdot x}$

e. $A = \frac{1}{2}\begin{pmatrix} -\sqrt{3} & 1 \\ 1 & \sqrt{3} \end{pmatrix} =$ $\boxed{\text{reflection about line } L:\ y = \sqrt{7+4\sqrt{3}}\, x}$

Part II. Give details. [40%]

f. Define reflection in a line L in $\mathcal{R}^3$. $A = \text{diag}(2u_1\vec{u}, 2u_2\vec{u}, 2u_3\vec{u}) - I$

$\boxed{T(\vec{x}) = 2(\vec{x}\cdot\vec{u})\vec{u} - \vec{x}}$

g. Display the matrix A of a projection onto a line L in $\mathcal{R}^3$.

g) $A = \text{aug}(u_1\vec{u}, u_2\vec{u}, u_3\vec{u})$

$$\boxed{A = \begin{bmatrix} u_1^2 & u_1u_2 & u_1u_3 \\ u_1u_2 & u_2^2 & u_2u_3 \\ u_1u_3 & u_2u_3 & u_3^2 \end{bmatrix}}$$

h. Define rotation clockwise by angle θ in $\mathcal{R}^2$.

h) $T(\vec{x}) = A\vec{x}$

$$\boxed{A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}}$$

a.) check: $T\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 0 \\ 2 \end{pmatrix}$

c) $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} u_1^2 & u_1u_2 \\ u_1u_2 & u_2^2 \end{pmatrix}$

$\Rightarrow u_1 = 0$, $u_2 = \pm 1$ $\quad \begin{cases} x = 0 + 0\cdot t \\ y = 0 + 1\cdot t \end{cases}$

$x = 0$

See back →

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d) $\begin{pmatrix} 3/4 & \sqrt{3}/4 \\ \sqrt{3}/4 & 1/4 \end{pmatrix}$

$u_1^2 = 3/4 \Rightarrow u_1 = \sqrt{3}/2$

$u_2^2 = 1/4 \Rightarrow u_2 = 1/2$

$L: \begin{cases} x = 0 + \sqrt{3}/2 \cdot t \\ y = 0 + 1/2 \cdot t \end{cases}$

$t = 2y = \frac{2}{\sqrt{3}} x$

$y = \frac{\sqrt{3}}{3} x$

e) $\begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} = \begin{pmatrix} a & b \\ b & -a \end{pmatrix}$ $a^2+b^2=$

$\begin{cases} 2u_1^2 - 1 = -\sqrt{3}/2 \\ 2u_1u_2 = 1/2 \\ 2u_2^2 - 1 = \sqrt{3}/2 \end{cases}$

$2u_1^2 = \frac{2-\sqrt{3}}{2}$

$u_1^2 = \frac{2-\sqrt{3}}{4}$

$u_1 = \frac{1}{2}\sqrt{2-\sqrt{3}}$

$2u_2^2 - 1 = \sqrt{3}/2$

$2u_2^2 = \frac{\sqrt{3}+2}{2}$

$u_2 = \frac{1}{2}\sqrt{2+\sqrt{3}}$

check: $\cancel{2}\left(\frac{1}{\cancel{2}}\sqrt{2-\sqrt{3}}\right)\left(\frac{1}{2}\sqrt{2+\sqrt{3}}\right)$

$\frac{1}{2}\sqrt{2-\sqrt{3}}\sqrt{2+\sqrt{3}}$

$\frac{1}{2}\sqrt{4-3}$

$= 1/2$

$\begin{cases} x = 0 + \frac{1}{2}\sqrt{2-\sqrt{3}} \cdot t \\ y = 0 + \frac{1}{2}\sqrt{2+\sqrt{3}} \cdot t \end{cases}$

$t = \frac{\cancel{2}x}{\sqrt{2-\sqrt{3}}} = \frac{\cancel{2}y}{\sqrt{2+\sqrt{3}}}$

$x\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}} = y$

$x\sqrt{\frac{(2+\sqrt{3})(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}} = y$

$x\sqrt{4+4\sqrt{3}+3} = y$

$y = \sqrt{7+4\sqrt{3}}\, x$

$y = \frac{\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}} x$

$y = \sqrt{\frac{(2+\sqrt{3})(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}}\, x$

$y = \sqrt{(2+\sqrt{3})^2}\, x$

$\boxed{y = (2+\sqrt{3})x}$