Math 2270 Extra Credit Problems
Chapter 2
August 2007

Due date: See the internet due dates. Records are locked on that date and only corrected, never appended.

Submitted work. Please submit one stapled package per problem. Kindly label problems [Extra Credit]. Label each problem with its corresponding problem number. You may attach this printed sheet to simplify your work.

Problem XC2.1-16. (Invertible T)
Decide invertibility of $T(x) = Ax$ for the following matrices $A$. Then find the matrix of $T^{-1}$, in each case.

\[
\begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}.
\]

Problem XC2.1-43. (Matrix of T)
(a) Suppose $v$ has components 2, $-2$, 5. Find the matrix of $T(x) = v \cdot x$.
(b) Prove that every linear transformation $T$ from $\mathbb{R}^3$ into $\mathbb{R}^1$ can be written as $T(x) = v \cdot x$ for some vector $v$.

Problem XC2.1-46. (Matrix of T)
(a) Let $T(x) = B(A(x))$ where

\[
A = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}.
\]

Find the matrix of $T$.
(b) What is the matrix of $T$ defined by $T(x) = B^2(A(x))$?

Problem XC2.2-18. (Reflection line equation)
Let a reflection $T$ have matrix $\frac{1}{2} \begin{pmatrix} \sqrt{3} & 1 \\ 1 & -\sqrt{3} \end{pmatrix}$. Find the equation for the line $L$ of reflection.

Problem XC2.2-26. (Matrix of T)
(a) Find the scaling matrix $A$ if $T\left(\begin{pmatrix} 2 \\ -1 \end{pmatrix}\right) = \begin{pmatrix} 8 \\ -4 \end{pmatrix}$.
(b) Find the projection matrix $A$ if $T\left(\begin{pmatrix} 2 \\ 3 \end{pmatrix}\right) = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$.

Problem XC2.2-39. (Composite linear transformations)
Each of the matrices below is a standard geometric linear transformation followed by a scaling. Find the scale factor.

\[
\begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}, \quad \begin{pmatrix} 6 & 0 \\ -2 & 6 \end{pmatrix}, \quad \begin{pmatrix} 3/4 & 1 \\ 1 & -3/4 \end{pmatrix}.
\]

End of extra credit problems chapter 2.