

Math 2270 Extra Credit Problems
Chapter 1
August 2007

Due date: See the internet due dates. Records are locked on that date and only corrected, never appended.

Submitted work. Please submit one stapled package per problem. Kindly label problems Extra Credit. Label each problem with its corresponding problem number. You may attach this printed sheet to simplify your work.

Problem XC1.1-26. (Three possibilities)

Determine which values of k correspond to (a) a unique solution, (b) no solution or (c) infinitely many solutions.

$$\begin{cases} x + 2y + kz = 0 \\ 2x + 4y + (k+1)z = 2 \\ 3x + 6y + (2k+1)z = 2 \end{cases}$$

Problem XC1.1-30. (Polynomial interpolation)

Find the polynomial $f(x) = a + bx + cx^2$ which passes through the points $(1, 9)$, $(2, 24)$, $(3, 47)$.

Problem XC1.1-32. (Polynomial interpolation)

Find all polynomials $f(x) = a + bx + cx^2$ which pass through the points $(1, 10)$, $(2, 28)$ and $f'(3) = 33$.

Problem XC1.2-22. (RREF)

Report five types of 3×4 matrices in RREF form.

Problem XC1.2-28. (Combo rule)

Consider the following systems.

$$(1) \quad \begin{cases} a_{11}x_1 + \cdots + a_{1n}x_n = b_1 \\ \vdots \\ a_{m1}x_1 + \cdots + a_{mn}x_n = b_m \end{cases}$$

$$(2) \quad \begin{cases} a_{11}x_1 + \cdots + a_{1n}x_n = b_1 \\ \vdots \\ c_{k1}x_1 + \cdots + c_{kn}x_n = d_k \\ \vdots \\ a_{m1}x_1 + \cdots + a_{mn}x_n = b_m \end{cases}$$

We assume the systems identical except for equation k , which is obtained in the second system by applying a combination rule $\text{combo}(\mathbf{r}, \mathbf{k}, \mathbf{c})$ to the first system. We assume $r \neq k$ and then the coefficients in the second system are given by

$$c_{kj} = a_{kj} + ca_{rj}, \quad j = 1, \dots, n, \quad d_k = b_k + cb_r.$$

(a) Prove that every solution of system (1) is a solution of system (2).

(b) Prove that every solution of system (2) is a solution of system (1).

Problem XC1.2-30. (Polynomial interpolation)

Find the polynomial $f(x) = a + bx + cx^2 + dx^3$ which satisfies $f(1) = 8$, $f(2) = 24$, $f(3) = 24$, $f(4) = 110$.

Problem XC1.3-26. (Matrix algebra)

Find a 3×3 matrix A which satisfies the following relations.

$$A \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad A \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad A \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

End of extra credit problems chapter 1.