

**Math 2250 Extra Credit Problems**  
**Chapter 10**  
**October 2007**

**Due date:** Submit these problems under the door, 113 JWB, by 9pm on the first day of final week. Records are locked on that date and only corrected, never appended. The scores can replace *any* missing score for the entire semester.

**Submitted work.** Please submit one stapled package per problem. Kindly label problems **Extra Credit**. Label each problem with its corresponding problem number, e.g., Xc10.3-20. You may attach this printed sheet to simplify your work.

**Problem Xc10.3-20. (Inverse transform)**

Solve for  $f(t)$  in the relation  $\mathcal{L}(f(t)) = \frac{1}{s^4 - 8s^2 + 16}$ . Use partial fractions in the details.

**Problem Xc10.3-24. (Inverse transform)**

Solve for  $f(t)$  in the relation  $\mathcal{L}(f(t)) = \frac{s}{s^4 + 4a^4}$ , showing the details that give the answer  $f(t) = \frac{1}{2a^2} \sinh at \sin at$

**Problem Xc10.4-12. (Inverse transform, convolution)**

Solve for  $f(t)$  in the relation  $\mathcal{L}(f(t)) = \frac{1}{s(s^2 + 4s + 5)}$ . Instead of the convolution theorem, use partial fractions for the details. If you can see how, then check the answer with the convolution theorem.

**Problem Xc10.4-26. (Inverse transform techniques)**

Use the  $s$ -differentiation theorem in the details of solving for  $f(t)$  in the relation  $\mathcal{L}(f(t)) = \arctan \frac{3}{s+2}$ . You will need to apply the theorem  $\lim_{s \rightarrow \infty} \mathcal{L}(f(t)) = 0$ .

**Problem Xc10.4-40. (Series methods for transforms)**

Expand in a series, using Taylor series formulas, the function  $f(t) = \frac{\cos 2\sqrt{t}}{\sqrt{\pi t}}$ . Then find  $\mathcal{L}(f(t))$  as a series by Laplace transform of each series term, separately. Finally, re-constitute the series in variable  $s$  into elementary functions, namely  $e^{-1/s}$  divided by  $\sqrt{s}$ .

**Problem Xc10.5-6. (Second shifting theorem, Heaviside step)**

Find the function  $f(t)$  in the relation  $\mathcal{L}(f(t)) = \frac{se^{-s}}{s^2 + \pi^2}$ .

**Problem Xc10.5-14. (Transforms of piecewise functions)**

Let  $f(t) = \begin{cases} \cos \pi t & 0 \leq t \leq 2, \\ 0 & t > 2. \end{cases}$  Find  $\mathcal{L}(f(t))$ . Details should expand  $f(t)$  as a linear combination of Heaviside step functions.

**Problem Xc10.5-26. (Sawtooth wave)**

Let  $f(t+a) = f(t)$  and  $f(t) = t$  on  $0 \leq t \leq a$ . Then  $f$  is  $a$ -periodic and has a Laplace transform obtained from the periodic function formula. Show the details in the derivation to obtain the answer  $\mathcal{L}(f(t)) = \frac{1}{as^2} - \frac{e^{-as}}{s(1 - e^{-as})}$ .

**Problem Xc10.5-28. (Modified sawtooth wave)**

Let  $f(t+2a) = f(t)$  and  $f(t) = t$  on  $0 \leq t \leq a$ ,  $f(t) = 0$  on  $a < t \leq 2a$ . Then  $f$  is  $2a$ -periodic and has a Laplace transform obtained from the periodic function formula. Derive a formula for  $\mathcal{L}(f(t))$ .

**End of extra credit problems chapter 10.**