Name. ________________________

Differential Equations 5410-1
Midterm Exam 4, Due classtime 27-Nov-2002

Scores

__________ **Problem 1**. Coupled spring-mass system
__________ **Problem 2**. Laplace transform
__________ **Problem 3**. Laplace inverse transform
__________ **Problem 4**. Dirac delta function
__________ **Average**.

Instructions. The four take-home problems constitute the entire exam. Answer checks are expected. If `maple` assist is used, then please attach the `maple` output.

1. **(Coupled spring-mass system)** The system

$$\begin{aligned} m_1 x_1'' &= -k_1 x_1 + k_2(x_2 - x_1), \\ m_2 x_2'' &= -k_2(x_2 - x_1) + k_3(x_3 - x_2), \\ m_3 x_3'' &= -k_3(x_3 - x_2) - k_4 x_3 \end{aligned} \tag{1}$$

represents three masses m_1, m_2, m_3 coupled by springs of Hooke's constant k_1, k_2, k_3. Let $m_1 = m_2 = m_3 = 1$, $k_1 = k_2 = k_3 = 1$. Find the natural frequencies ω_1, ω_2, ω_3 of ocillation of system (1). Find a matrix formula for x_1, x_2, x_3 involving the eigenpairs of the coefficient matrix of system (1). Check the answer using `maple`; see `exponential`.

2. **(Laplace transform)** Solve $x'' + x = \cos 2t$, $x(0) = 0$, $x'(0) = 0$ by two methods: (1) Undetermined coefficients and (2) Laplace transform. Show all steps, thus verifying the *maple* answer

$$x(t) = -\frac{1}{3}\cos(2t) + \frac{1}{3}\cos(t).$$

3. **(Laplace inverse transform)** Show the partial fraction steps involved in solving for $f(t)$ in the Laplace equation

$$\mathcal{L}(f(t)) = \frac{2s}{(s-1)^2(s-2)(s^2+1)}.$$

Kindly flag the step where Lerch's theorem is applied, to justify the *maple* answer

$$f(t) = -te^t - e^t + \frac{4}{5}e^{2t} + \frac{1}{5}\cos(t) + \frac{2}{5}\sin(t).$$

4. **(Delta function)** Show the Laplace steps in solving the hammer-hit oscillator problem $x'' + x = 10\delta(t-1)$, $x(0) = 0$, $x'(0) = 1$. The *maple* answer is $x(t) = 10H(t-1)\sin(t-1) + \sin(t)$, $H =$ Heaviside's unit step.