Name.

Differential Equations 5410-1 Midterm Exam 3, Due classtime 15-Nov-2002

Scores

Problem 1	1 . Variation of Parameters
Problem 2	2. Undetermined Coefficients
Problem 3	3 . Superposition
Problem 4	4. Mechanical system
Average.	·

Instructions. The four take-home problems constitute the entire exam. Answer checks are expected. If maple assist is used, then please attach the maple output.

1. (Variation of Parameters) Show the steps in the solution of

$$y'' + y = xe^x$$

by variation of parameters, to obtain the general solution

$$y = c_1 \cos x + c_2 \sin x + \frac{1}{2}(x-1)e^x.$$

2. (Undetermined Coefficients) Show the steps in the solution of

$$y''' - y' = x^2 + xe^x + \cos x$$

by undetermined coefficients, to obtain the general solution

$$y = c_1 e^x + c_2 e^{-x} + c_3 + \frac{1}{4} (x^2 - 3x + 7/2)e^x - \frac{1}{3}x^3 - 2x - \frac{1}{2}\sin x.$$

3. (Superposition) Determine via superposition the function f(t) and three values m, c, k such that the general solution of mx'' + cx' + kx = f(t) is given by the formula

$$x = c_1 e^{-t} \cos(\omega t) + c_2 e^{-t} \sin(\omega t) + t e^{-2t} \sin 4t.$$

The frequency ω is assumed positive.

4. (Mechanical System) Find the constant A such that $y_p = 60 \cos t + 2 \sin t$ is a particular solution of the system $16y'' + 8y' + 256y = A \cos t$. Find the general solution $y = y_h + y_p$. Discuss the possibility of a second periodic solution to this system, mathematically and physically.