

Name. \_\_\_\_\_

Section. \_\_\_\_\_

**Differential Equations 5410**  
**Midterm Exam 1, Fall 2002**  
**Exam Date: Monday, 16 September, 2002**

**Instructions.** The four problems below are take-home, due on the date above. Answer checks are expected. If `maple assist` is used, then please attach the `maple` output.

The remaining 20% of the exam is in class, 15 minutes, one problem, of a type similar to one of the problems below. Calculators, hand-written or computer-generated notes are allowed, including xerox copies of tables or classroom xerox notes. Books are not allowed.

**Scores**

\_\_\_\_\_ **Problem 1.** Separable Equations.  
\_\_\_\_\_ **Problem 2.** Linear Equations.  
\_\_\_\_\_ **Problem 3.** Planar systems.  
\_\_\_\_\_ **Problem 4.** Application.  
\_\_\_\_\_ **Problem 5.** In-class, Sept 16.  
\_\_\_\_\_ **Average.**

1. **(Separable Equations)** Solve the separable problem for the *implicit* and *explicit* solutions. Distinguish equilibrium and non-equilibrium solutions as needed.

$$2y' = x \cos x + \cot x - 4(y + 1)^2(x + \csc x) \cos x.$$

2. **(Linear Equations)** Solve the linear equation  $2xy'(x) + 5y(x) = \sqrt{x}e^{-2x}$ ,  $y(1) = 5$ . Expected details include the factorization method and all integration steps (by hand).
3. **(Planar systems)** State and prove a result for planar autonomous systems that parallels the first order result: Solutions of  $y' = f(y)$  exist, they are uniquely determined by initial data, and solutions don't cross, provided  $f$  is of class  $C^1$ .
4. **(Application: Torricelli's law)** A water tank has the shape of  $y = x^{6/5}$  revolved around the  $y$ -axis. The depth of the water is 20 feet. After 1 hour and 15 minutes, the drain hole at  $(0, 0)$  empties the tank to a depth of 12 feet. Find the additional time it takes to empty the tank. Ref: Edwards-Penney, Exercise 1.4-52 and equation 1.4-(24).