

Differential Equations 2280
Sample Midterm Exam 3
Thursday, 27 April 2006

Instructions: This in-class exam is 50 minutes. No calculators, notes, tables or books. No answer check is expected. Details count 75%. The answer counts 25%.

1. (ch6)

- (a) Define *asymptotically stable equilibrium* for $\mathbf{u}' = \mathbf{f}(\mathbf{u})$, a 2-dimensional system.
- (b) Give examples of 2-dimensional systems of type saddle, spiral, center and node.
- (c) Give a 2-dimensional predator-prey example $\mathbf{u}' = \mathbf{f}(\mathbf{u})$ and explain the meaning of the variables in the model.

2. (ch6)

Find the equilibrium points of $x' = 14x - x^2/2 - xy$, $y' = 16y - y^2/2 - xy$ and classify the linearizations as node, spiral, center, saddle. What classifications can be deduced for the nonlinear system?

3. (ch7)

- (a) Define the direct Laplace Transform.
- (b) Define Heaviside's unit step function.
- (c) Derive a Laplace integral formula for Heaviside's unit step function.
- (d) Explain Laplace's Method, as applied to the differential equation $x'(t) + 2x(t) = e^t$, $x(0) = 1$.

4. (ch7)

- (a) Solve $\mathcal{L}(f(t)) = \frac{100}{s^2 + 1}(s^2 + 4)$ for $f(t)$.
- (b) Solve for $f(t)$ in the equation $\mathcal{L}(f(t)) = \frac{1}{s^2(s - 3)}$.
- (c) Find $\mathcal{L}(f)$ given $f(t) = (-t)e^{2t} \sin(3t)$.
- (d) Find $\mathcal{L}(f)$ where $f(t)$ is the periodic sawtooth wave function of period 2 equal to $t/2$ on $0 \leq t \leq 2$.

5. (ch7)

- (a) Solve $y'' + 4y' + 4y = t^2$, $y(0) = y'(0) = 0$ by Laplace's Method.
- (b) Solve $x''' + x'' - 6x' = 0$, $x(0) = x'(0) = 0$, $x''(0) = 1$ by Laplace's Method.
- (c) Solve the system $x' = x + y$, $y' = x - y + e^t$, $x(0) = 0$, $y(0) = 0$ by Laplace's Method.