Differential Equations 2280
Sample Midterm Exam 3
Thursday, 27 April 2006

Instructions: This in-class exam is 50 minutes. No calculators, notes, tables or books. No answer check is expected. Details count 75%. The answer counts 25%.

1. (ch6)
   (a) Define asymptotically stable equilibrium for $u' = f(u)$, a 2-dimensional system.
   (b) Give examples of 2-dimensional systems of type saddle, spiral, center and node.
   (c) Give a 2-dimensional predator-prey example $u' = f(u)$ and explain the meaning of the variables in the model.

2. (ch6)
   Find the equilibrium points of $x' = 14x - x^2/2 - xy$, $y' = 16y - y^2/2 - xy$ and classify the linearizations as node, spiral, center, saddle. What classifications can be deduced for the nonlinear system?

3. (ch7)
   (a) Define the direct Laplace Transform.
   (b) Define Heaviside’s unit step function.
   (c) Derive a Laplace integral formula for Heaviside’s unit step function.
   (d) Explain Laplace’s Method, as applied to the differential equation $x'(t) + 2x(t) = e^t$, $x(0) = 1$.

4. (ch7)
   (a) Solve $\mathcal{L}(f(t)) = \frac{100}{s^2 + 1)(s^2 + 4)}$ for $f(t)$.
   (b) Solve for $f(t)$ in the equation $\mathcal{L}(f(t)) = \frac{1}{s^2(s - 3)}$.
   (c) Find $\mathcal{L}(f)$ given $f(t) = (-t)e^{2t} \sin(3t)$.
   (d) Find $\mathcal{L}(f)$ where $f(t)$ is the periodic sawtooth wave function of period 2 equal to $t/2$ on $0 \leq t \leq 2$.

5. (ch7)
   (a) Solve $y'' + 4y' + 4y = t^2$, $y(0) = y'(0) = 0$ by Laplace’s Method.
   (b) Solve $x'' + x' - 6x' = 0$, $x(0) = x'(0) = 0$, $x''(0) = 1$ by Laplace’s Method.
   (c) Solve the system $x' = x + y$, $y' = x - y + e^t$, $x(0) = 0$, $y(0) = 0$ by Laplace’s Method.